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Strong Arithmetic of Real Numbers

Andrzej Trybulec
Warsaw University
Białystok

Summary. This abstract contains the second part of the axiomatics of the Mizar system (the first part is in abstract [4]). The axioms listed here characterize the Mizar built-in concepts that are automatically attached to every Mizar article. We give definitional axioms of the following concepts: element, subset, Cartesian product, domain (non empty subset), subdomain (non empty subset of a domain), set domain (domain consisting of sets). Axioms of strong arithmetics of real numbers are also included.

MML Identifier: AXIOMS.

WWW: http://mizar.org/JFM/Addenda/axioms.html

The articles [4], [3], [6], [1], [2], and [5] provide the notation and terminology for this paper.

We adopt the following rules: x, y, z are real numbers, k is a natural number, and i is an element of $\mathbb{N}$.

One can prove the following propositions:

(19)[1] There exists y such that $x+y=0$.

(20) If $x \neq 0$, then there exists y such that $x \cdot y = 1$.

(21) If $x \leq y$ and $y \leq x$, then $x = y$.

(22) If $x \leq y$ and $y \leq z$, then $x \leq z$.

(24)[2] If $x \leq y$, then $x+z \leq y+z$.

(25) If $x \leq y$ and $0 \leq z$, then $x \cdot z \leq y \cdot z$.

(26) Let X, Y be subsets of $\mathbb{R}$. Suppose that for all x, y such that $x \in X$ and $y \in Y$ holds $x \leq y$. Then there exists z such that for all x, y such that $x \in X$ and $y \in Y$ holds $x \leq z$ and $z \leq y$.

(28)[3] If $x \in \mathbb{N}$ and $y \in \mathbb{N}$, then $x+y \in \mathbb{N}$.

(29) For every subset A of $\mathbb{R}$ such that $0 \in A$ and for every x such that $x \in A$ holds $x+1 \in A$ holds $\mathbb{N} \subseteq A$.

(30) $k = \{i : i < k\}$.

[1] The propositions (1)–(18) have been removed.
[2] The proposition (23) has been removed.
[3] The proposition (27) has been removed.

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