JOURNAL OF FORMALIZED MATHEMATICS
Volume 5, Released 1993, Published 2003
Inst. of Computer Science, Univ. of Białystok

Basic Concepts for Petri Nets with Boolean Markings

Pauline N. Kawamoto
Shinshu University
Nagano

Yasushi Fuwa
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. Contains basic concepts for Petri nets with Boolean markings and the firability/firing of single transitions as well as sequences of transitions [6]. The concept of a Boolean marking is introduced as a mapping of a Boolean TRUE/FALSE to each of the places in a place/transition net. This simplifies the conventional definitions of the firability and firing of a transition. One note of caution in this article - the definition of firing a transition does not require that the transition be firable. Therefore, it is advisable to check that transitions ARE firable before firing them.

MML Identifier: BOOLMARK.

WWW: http://mizar.org/JFM/Vol5/boolmark.html

The articles [10], [13], [1], [14], [3], [4], [9], [11], [8], [2], [12], [5], [15], and [7] provide the notation and terminology for this paper.

1. Preliminaries

The following three propositions are true:

(1) Let A, B be non empty sets, f be a function from A into B, C be a subset of A, and v be an element of B. Then $f {+}\cdot (C \longmapsto v)$ is a function from A into B.

(2) Let X, Y be non empty sets, A, B be subsets of X, and f be a function from X into Y. If $f^\circ A$ misses $f^\circ B$, then A misses B.

(3) For all sets A, B and for every function f and for every set x such that A misses B holds $(f {+}\cdot (A \longmapsto x))^\circ B = f^\circ B$.

2. Boolean Marking and Firability/Firing of Transitions

Let P_1 be a place/transition net structure. The functor Bool_marks_ofP_1 yields a non empty set of functions from the places of P_1 to *Boolean* and is defined as follows:

(Def. 1) Bool_marks_of$P_1 = \mathit{Boolean}^{\text{the places of } P_1}$.

Let P_1 be a place/transition net structure. A Boolean marking of P_1 is an element of Bool_marks_ofP_1.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. We say that t is firable on M_0 if and only if:

(Def. 2) $M_0{}^\circ({}^*\{t\}) \subseteq \{\mathit{true}\}$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. The functor Firing(t, M_0) yields a Boolean marking of P_1 and is defined by:

(Def. 3) $\mathrm{Firing}(t, M_0) = M_0 +\cdot ({}^*\{t\} \longmapsto \mathit{false}) +\cdot (\overline{\{t\}} \longmapsto \mathit{true})$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. We say that Q is firable on M_0 if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) $Q = \emptyset$, or

(ii) there exists a finite sequence M of elements of Bool_marks_ofP_1 such that $\mathrm{len}\, Q = \mathrm{len}\, M$ and Q_1 is firable on M_0 and $M_1 = \mathrm{Firing}(Q_1, M_0)$ and for every natural number i such that $i < \mathrm{len}\, Q$ and $i > 0$ holds Q_{i+1} is firable on M_i and $M_{i+1} = \mathrm{Firing}(Q_{i+1}, M_i)$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. The functor $\mathrm{Firing}(Q, M_0)$ yields a Boolean marking of P_1 and is defined as follows:

(Def. 5)(i) $\mathrm{Firing}(Q, M_0) = M_0$ if $Q = \emptyset$,

(ii) there exists a finite sequence M of elements of Bool_marks_ofP_1 such that $\mathrm{len}\, Q = \mathrm{len}\, M$ and $\mathrm{Firing}(Q, M_0) = M_{\mathrm{len}\, M}$ and $M_1 = \mathrm{Firing}(Q_1, M_0)$ and for every natural number i such that $i < \mathrm{len}\, Q$ and $i > 0$ holds $M_{i+1} = \mathrm{Firing}(Q_{i+1}, M_i)$, otherwise.

We now state several propositions:

(5)[1] For every non empty set A and for every set y and for every function f holds $(f +\cdot (A \longmapsto y))^\circ A = \{y\}$.

(6) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, t be a transition of P_1, and s be a place of P_1. If $s \in \overline{\{t\}}$, then $(\mathrm{Firing}(t, M_0))(s) = \mathit{true}$.

(7) Let P_1 be a place/transition net structure and S_1 be a non empty subset of the places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0{}^\circ S_1 = \{\mathit{false}\}$ and for every transition t of P_1 such that t is firable on M_0 holds $(\mathrm{Firing}(t, M_0))^\circ S_1 = \{\mathit{false}\}$.

(8) Let D be a non empty set, Q_0, Q_1 be finite sequences of elements of D, and i be a natural number. If $1 \leq i$ and $i \leq \mathrm{len}\, Q_0$, then $(Q_0 \frown Q_1)_i = (Q_0)_i$.

(10)[2] Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and Q_0, Q_1 be finite sequences of elements of the transitions of P_1. Then $\mathrm{Firing}(Q_0 \frown Q_1, M_0) = \mathrm{Firing}(Q_1, \mathrm{Firing}(Q_0, M_0))$.

(11) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and Q_0, Q_1 be finite sequences of elements of the transitions of P_1. If $Q_0 \frown Q_1$ is firable on M_0, then Q_1 is firable on $\mathrm{Firing}(Q_0, M_0)$ and Q_0 is firable on M_0.

(12) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and t be a transition of P_1. Then t is firable on M_0 if and only if $\langle t \rangle$ is firable on M_0.

(13) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and t be a transition of P_1. Then $\mathrm{Firing}(t, M_0) = \mathrm{Firing}(\langle t \rangle, M_0)$.

(14) Let P_1 be a place/transition net structure and S_1 be a non empty subset of the places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0{}^\circ S_1 = \{\mathit{false}\}$ and for every finite sequence Q of elements of the transitions of P_1 such that Q is firable on M_0 holds $(\mathrm{Firing}(Q, M_0))^\circ S_1 = \{\mathit{false}\}$.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Andrzej Trybulec for his patience and guidance in the writing of this article.

[1] The proposition (4) has been removed.
[2] The proposition (9) has been removed.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[3] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[4] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_2.html`.

[5] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/funct_4.html`.

[6] Pauline N. Kawamoto, Masayoshi Eguchi, Yasushi Fuwa, and Yatsuka Nakamura. The detection of deadlocks in Petri nets with ordered evaluation sequences. In *Institute of Electronics, Information, and Communication Engineers (IEICE) Technical Report*, pages 45–52. Institute of Electronics, Information, and Communication Engineers (IEICE), January 1993.

[7] Pauline N. Kawamoto, Yasushi Fuwa, and Yatsuka Nakamura. Basic Petri net concepts. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/petri.html`.

[8] Andrzej Trybulec. Binary operations applied to functions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funcop_1.html`.

[9] Andrzej Trybulec. Domains and their Cartesian products. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/domain_1.html`.

[10] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[11] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/fraenkel.html`.

[12] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finseq_4.html`.

[13] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[14] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

[15] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/margrel1.html`.

Received October 8, 1993

Published January 2, 2004