JOURNAL OF FORMALIZED MATHEMATICS
Volume 12, Released 2000, Published 2003
Inst. of Computer Science, Univ. of Białystok

The Concept of Fuzzy Relation and Basic Properties of its Operation

Takashi Mitsuishi
Shinshu University
Nagano

Katsumi Wasaki
Shinshu University
Nagano

Yasunari Shidama
Shinshu University
Nagano

Summary. This article introduces the fuzzy relation. This is the expansion of usual relation, and the value is given at the fuzzy value. At first, the definition of the fuzzy relation characterized by membership function is described. Next, the definitions of the zero relation and universe relation and basic operations of these relations are shown.

MML Identifier: FUZZY_3.

WWW: http://mizar.org/JFM/Vol12/fuzzy_3.html

The articles [2], [5], [6], [1], [3], and [4] provide the notation and terminology for this paper.

1. DEFINITION OF FUZZY RELATION

In this paper C_1, C_2 are non empty sets.

Let C be a non empty set. One can verify that every membership function of C is quasi total.

Let C_1, C_2 be non empty sets. A membership function of C_1, C_2 is a membership function of $[:C_1, C_2:]$.

Let C_1, C_2 be non empty sets and let h be a membership function of C_1, C_2. A fuzzy relation of C_1, C_2, h is a FuzzySet of $[:C_1, C_2:]$, h.

In the sequel f, g denote membership functions of C_1, C_2.

2. ZERO RELATION AND UNIVERSE RELATION

Let C_1, C_2 be non empty sets. A zero relation of C_1, C_2 is an Empty FuzzySet of $[:C_1, C_2:]$. A universe relation of C_1, C_2 is a Universal FuzzySet of $[:C_1, C_2:]$.

In the sequel X denotes a universe relation of C_1, C_2 and O denotes a zero relation of C_1, C_2.

Let C_1, C_2 be non empty sets. The functor $\mathrm{Zmf}(C_1,C_2)$ yielding a membership function of C_1, C_2 is defined by:

(Def. 1) $\mathrm{Zmf}(C_1,C_2) = \chi_{\emptyset,[:C_1,C_2:]}$.

The functor $\mathrm{Umf}(C_1,C_2)$ yielding a membership function of C_1, C_2 is defined as follows:

(Def. 2) $\mathrm{Umf}(C_1,C_2) = \chi_{[:C_1,C_2:],[:C_1,C_2:]}$.

We now state several propositions:

(45)[1] $\mathrm{Zmf}(C_1,C_2) = \mathrm{EMF}[:C_1, C_2:]$.

[1] The propositions (1)–(44) have been removed.

(46) $\mathrm{Umf}(C_1,C_2) = \mathrm{UMF}[:C_1, C_2:]$.

(47) O is a fuzzy relation of C_1, C_2, $\mathrm{Zmf}(C_1,C_2)$.

(48) X is a fuzzy relation of C_1, C_2, $\mathrm{Umf}(C_1,C_2)$.

(52)[2] For every element x of $[:C_1, C_2:]$ and for every membership function h of C_1, C_2 holds $(\mathrm{Zmf}(C_1,C_2))(x) \leq h(x)$ and $h(x) \leq (\mathrm{Umf}(C_1,C_2))(x)$.

(53) $\max(f,\mathrm{Umf}(C_1,C_2)) = \mathrm{Umf}(C_1,C_2)$ and $\min(f,\mathrm{Umf}(C_1,C_2)) = f$ and $\max(f,\mathrm{Zmf}(C_1,C_2)) = f$ and $\min(f,\mathrm{Zmf}(C_1,C_2)) = \mathrm{Zmf}(C_1,C_2)$.

(61)[3] $1\text{-minus}\,\mathrm{Zmf}(C_1,C_2) = \mathrm{Umf}(C_1,C_2)$ and $1\text{-minus}\,\mathrm{Umf}(C_1,C_2) = \mathrm{Zmf}(C_1,C_2)$.

(121)[4] If $\min(f, 1\text{-minus}\,g) = \mathrm{Zmf}(C_1,C_2)$, then for every element c of $[:C_1, C_2:]$ holds $f(c) \leq g(c)$.

(123)[5] If $\min(f,g) = \mathrm{Zmf}(C_1,C_2)$, then $\min(f, 1\text{-minus}\,g) = f$.

REFERENCES

[1] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[2] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[3] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/rfunct_1.html.

[4] Takashi Mitsuishi, Noboru Endou, and Yasunari Shidama. The concept of fuzzy set and membership function and basic properties of fuzzy set operation. *Journal of Formalized Mathematics*, 12, 2000. http://mizar.org/JFM/Vol12/fuzzy_1.html.

[5] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[6] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.

Received September 15, 2000

Published January 2, 2004

[2] The propositions (49)–(51) have been removed.
[3] The propositions (54)–(60) have been removed.
[4] The propositions (62)–(120) have been removed.
[5] The proposition (122) has been removed.