JOURNAL OF FORMALIZED MATHEMATICS
Volume 1, Released 1989, Published 2003
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Graphs of Functions

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Summary. The graph of a function is defined in [1]. In this paper the graph of a function is redefined as a Relation. Operations on functions are interpreted as the corresponding operations on relations. Some theorems about graphs of functions are proved.

MML Identifier: GRFUNC_1.

WWW: http://mizar.org/JFM/Vol1/grfunc_1.html

The articles [2], [3], and [1] provide the notation and terminology for this paper.

We adopt the following convention: X, Y, x, x_1, x_2, y, y_1, y_2, z are sets and f, g, h are functions.

The following propositions are true:

(6)[1] For every set G such that $G \subseteq f$ holds G is a function.

(8)[2] $f \subseteq g$ iff $\operatorname{dom} f \subseteq \operatorname{dom} g$ and for every x such that $x \in \operatorname{dom} f$ holds $f(x) = g(x)$.

(9) If $\operatorname{dom} f = \operatorname{dom} g$ and $f \subseteq g$, then $f = g$.

(12)[3] If $\langle x, z \rangle \in g \cdot f$, then $\langle x, f(x) \rangle \in f$ and $\langle f(x), z \rangle \in g$.

(13) If $h \subseteq f$, then $g \cdot h \subseteq g \cdot f$ and $h \cdot g \subseteq f \cdot g$.

(15)[4] $\{\langle x, y \rangle\}$ is a function.

(16) If $f = \{\langle x, y \rangle\}$, then $f(x) = y$.

(18)[5] If $\operatorname{dom} f = \{x\}$, then $f = \{\langle x, f(x) \rangle\}$.

(19) $\{\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle\}$ is a function iff if $x_1 = x_2$, then $y_1 = y_2$.

(25)[6] f is one-to-one iff for all x_1, x_2, y such that $\langle x_1, y \rangle \in f$ and $\langle x_2, y \rangle \in f$ holds $x_1 = x_2$.

(26) If $g \subseteq f$ and f is one-to-one, then g is one-to-one.

(27) $f \cap X$ is a function and $X \cap f$ is a function.

(28) If $h = f \cap g$, then $\operatorname{dom} h \subseteq \operatorname{dom} f \cap \operatorname{dom} g$ and $\operatorname{rng} h \subseteq \operatorname{rng} f \cap \operatorname{rng} g$.

(29) If $h = f \cap g$ and $x \in \operatorname{dom} h$, then $h(x) = f(x)$ and $h(x) = g(x)$.

[1] The propositions (1)–(5) have been removed.
[2] The proposition (7) has been removed.
[3] The propositions (10) and (11) have been removed.
[4] The proposition (14) has been removed.
[5] The proposition (17) has been removed.
[6] The propositions (20)–(24) have been removed.

(30) If f is one-to-one or g is one-to-one and if $h = f \cap g$, then h is one-to-one.

(31) If $\mathrm{dom}\, f$ misses $\mathrm{dom}\, g$, then $f \cup g$ is a function.

(32) If $f \subseteq h$ and $g \subseteq h$, then $f \cup g$ is a function.

(33) If $h = f \cup g$, then $\mathrm{dom}\, h = \mathrm{dom}\, f \cup \mathrm{dom}\, g$ and $\mathrm{rng}\, h = \mathrm{rng}\, f \cup \mathrm{rng}\, g$.

(34) If $x \in \mathrm{dom}\, f$ and $h = f \cup g$, then $h(x) = f(x)$.

(35) If $x \in \mathrm{dom}\, g$ and $h = f \cup g$, then $h(x) = g(x)$.

(36) If $x \in \mathrm{dom}\, h$ and $h = f \cup g$, then $h(x) = f(x)$ or $h(x) = g(x)$.

(37) If f is one-to-one and g is one-to-one and $h = f \cup g$ and $\mathrm{rng}\, f$ misses $\mathrm{rng}\, g$, then h is one-to-one.

(38) $f \setminus X$ is a function.

(46)[7] If $f = \emptyset$, then f is one-to-one.

(47) If f is one-to-one, then for all x, y holds $\langle y, x \rangle \in f^{-1}$ iff $\langle x, y \rangle \in f$.

(49)[8] If $f = \emptyset$, then $f^{-1} = \emptyset$.

(52)[9] $x \in \mathrm{dom}\, f$ and $x \in X$ iff $\langle x, f(x) \rangle \in f \restriction X$.

(54)[10] $(f \restriction X) \cdot h \subseteq f \cdot h$ and $g \cdot (f \restriction X) \subseteq g \cdot f$.

(64)[11] If $g \subseteq f$, then $f \restriction \mathrm{dom}\, g = g$.

(67)[12] $x \in \mathrm{dom}\, f$ and $f(x) \in Y$ iff $\langle x, f(x) \rangle \in Y \restriction f$.

(69)[13] $(Y \restriction f) \cdot h \subseteq f \cdot h$ and $g \cdot (Y \restriction f) \subseteq g \cdot f$.

(79)[14] If $g \subseteq f$ and f is one-to-one, then $\mathrm{rng}\, g \restriction f = g$.

(87)[15] $x \in f^{-1}(Y)$ iff $\langle x, f(x) \rangle \in f$ and $f(x) \in Y$.

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Received April 14, 1989

Published January 2, 2004

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[7] The propositions (39)–(45) have been removed.
[8] The proposition (48) has been removed.
[9] The propositions (50) and (51) have been removed.
[10] The proposition (53) has been removed.
[11] The propositions (55)–(63) have been removed.
[12] The propositions (65) and (66) have been removed.
[13] The proposition (68) has been removed.
[14] The propositions (70)–(78) have been removed.
[15] The propositions (80)–(86) have been removed.