

Properties of the Upper and Lower Sequence on the Cage¹

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The articles [25], [29], [2], [27], [1], [4], [5], [3], [15], [13], [26], [23], [24], [12], [22], [9], [10], [7], [28], [16], [11], [19], [18], [6], [21], [20], [8], [14], and [17] provide the notation and terminology for this paper.

In this paper n is a natural number.

We now state a number of propositions:

- (1) For all subsets A, B of \mathcal{E}_T^2 such that A meets B holds $\text{proj1}^\circ A$ meets $\text{proj1}^\circ B$.
- (2) Let A, B be subsets of \mathcal{E}_T^2 and s be a real number. If A misses B and $A \subseteq \text{HorizontalLine}(s)$ and $B \subseteq \text{HorizontalLine}(s)$, then $\text{proj1}^\circ A$ misses $\text{proj1}^\circ B$.
- (3) For every closed subset S of \mathcal{E}_T^2 such that S is Bounded holds $\text{proj1}^\circ S$ is closed.
- (4) For every compact subset S of \mathcal{E}_T^2 holds $\text{proj1}^\circ S$ is compact.
- (5) Let p, q, p_1, q_1 be points of \mathcal{E}_T^2 . Suppose $\mathcal{L}(p, q)$ is vertical and $\mathcal{L}(p_1, q_1)$ is vertical and $p_1 = (p_1)_1$ and $p_2 \leq (p_1)_2$ and $(p_1)_2 \leq (q_1)_2$ and $(q_1)_2 \leq q_2$. Then $\mathcal{L}(p_1, q_1) \subseteq \mathcal{L}(p, q)$.
- (6) Let p, q, p_1, q_1 be points of \mathcal{E}_T^2 . Suppose $\mathcal{L}(p, q)$ is horizontal and $\mathcal{L}(p_1, q_1)$ is horizontal and $p_2 = (p_1)_2$ and $p_1 \leq (p_1)_1$ and $(p_1)_1 \leq (q_1)_1$ and $(q_1)_1 \leq q_1$. Then $\mathcal{L}(p_1, q_1) \subseteq \mathcal{L}(p, q)$.
- (7) Let G be a Go-board and i, j, k, j_1, k_1 be natural numbers. Suppose $1 \leq i$ and $i \leq \text{len } G$ and $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \text{width } G$. Then $\mathcal{L}(G \circ (i, j_1), G \circ (i, k_1)) \subseteq \mathcal{L}(G \circ (i, j), G \circ (i, k))$.
- (8) Let G be a Go-board and i, j, k, j_1, k_1 be natural numbers. Suppose $1 \leq i$ and $i \leq \text{width } G$ and $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \text{len } G$. Then $\mathcal{L}(G \circ (j_1, i), G \circ (k_1, i)) \subseteq \mathcal{L}(G \circ (j, i), G \circ (k, i))$.
- (9) Let G be a Go-board and j, k, j_1, k_1 be natural numbers. Suppose $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \text{width } G$. Then $\mathcal{L}(G \circ (\text{Center } G, j_1), G \circ (\text{Center } G, k_1)) \subseteq \mathcal{L}(G \circ (\text{Center } G, j), G \circ (\text{Center } G, k))$.
- (10) Let G be a Go-board. Suppose $\text{len } G = \text{width } G$. Let j, k, j_1, k_1 be natural numbers. Suppose $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \text{len } G$. Then $\mathcal{L}(G \circ (j_1, \text{Center } G), G \circ (k_1, \text{Center } G)) \subseteq \mathcal{L}(G \circ (j, \text{Center } G), G \circ (k, \text{Center } G))$.

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- (11) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, j_1)\}$.
- (12) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, k_1)\}$.
- (13) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, j_1)\}$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, k_1)\}$.
- (14) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{lenGauge}(C, n)$ and $1 \leq i$ and $i \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (j, i) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k, i)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (j_1, i)\}$.
- (15) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{lenGauge}(C, n)$ and $1 \leq i$ and $i \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (k, i) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (k_1, i)\}$.
- (16) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{lenGauge}(C, n)$ and $1 \leq i$ and $i \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (j, i) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (k, i) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (j_1, i)\}$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (k_1, i)\}$.
- (17) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, j_1)\}$.
- (18) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, k_1)\}$.
- (19) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \text{lenGauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{widthGauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exist natural numbers j_1, k_1 such

that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, j_1)\}$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j_1), \text{Gauge}(C, n) \circ (i, k_1)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i, k_1)\}$.

- (20) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{len Gauge}(C, n)$ and $1 \leq i$ and $i \leq \text{width Gauge}(C, n)$ and $\text{Gauge}(C, n) \circ (j, i) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k, i)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (j_1, i)\}$.
- (21) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{len Gauge}(C, n)$ and $1 \leq i$ and $i \leq \text{width Gauge}(C, n)$ and $\text{Gauge}(C, n) \circ (k, i) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (k_1, i)\}$.
- (22) Let C be a compact connected non vertical non horizontal subset of \mathcal{E}_T^2 and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{len Gauge}(C, n)$ and $1 \leq i$ and $i \leq \text{width Gauge}(C, n)$ and $\text{Gauge}(C, n) \circ (j, i) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (k, i) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (j_1, i)\}$ and $\mathcal{L}(\text{Gauge}(C, n) \circ (j_1, i), \text{Gauge}(C, n) \circ (k_1, i)) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (k_1, i)\}$.
- (23) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k))$ meets $\text{LowerArc}(C)$.
- (24) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n)$ and $\text{Gauge}(C, n) \circ (i, k) \in \tilde{\mathcal{L}}(\text{UpperSeq}(C, n))$ and $\text{Gauge}(C, n) \circ (i, j) \in \tilde{\mathcal{L}}(\text{LowerSeq}(C, n))$. Then $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k))$ meets $\text{UpperArc}(C)$.
- (25) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n)$ and $n > 0$ and $\text{Gauge}(C, n) \circ (i, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ and $\text{Gauge}(C, n) \circ (i, j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$. Then $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k))$ meets $\text{LowerArc}(C)$.
- (26) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n)$ and $n > 0$ and $\text{Gauge}(C, n) \circ (i, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ and $\text{Gauge}(C, n) \circ (i, j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$. Then $\mathcal{L}(\text{Gauge}(C, n) \circ (i, j), \text{Gauge}(C, n) \circ (i, k))$ meets $\text{UpperArc}(C)$.
- (27) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n+1)$ and $\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n+1)))$ and $\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n+1)))$. Then $\mathcal{L}(\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), j), \text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), k))$ meets $\text{LowerArc}(C)$.
- (28) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n+1)$ and $\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n+1)))$ and $\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n+1)))$. Then $\mathcal{L}(\text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), j), \text{Gauge}(C, n+1) \circ (\text{Center Gauge}(C, n+1), k))$ meets $\text{UpperArc}(C)$.

- (49) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_2$ and $i_2 \leq i_1$ and $i_1 < \text{len Gauge}(C, n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n)$ and $(\mathcal{L}(\text{Gauge}(C, n) \circ (i_1, j), \text{Gauge}(C, n) \circ (i_1, k)) \cup \mathcal{L}(\text{Gauge}(C, n) \circ (i_1, k), \text{Gauge}(C, n) \circ (i_2, k))) \cap \tilde{\mathcal{L}}(\text{UpperSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i_2, k)\}$ and $(\mathcal{L}(\text{Gauge}(C, n) \circ (i_1, j), \text{Gauge}(C, n) \circ (i_1, k)) \cup \mathcal{L}(\text{Gauge}(C, n) \circ (i_1, k), \text{Gauge}(C, n) \circ (i_2, k))) \cap \tilde{\mathcal{L}}(\text{LowerSeq}(C, n)) = \{\text{Gauge}(C, n) \circ (i_1, j)\}$. Then $\mathcal{L}(\text{Gauge}(C, n) \circ (i_1, j), \text{Gauge}(C, n) \circ (i_1, k)) \cup \mathcal{L}(\text{Gauge}(C, n) \circ (i_1, k), \text{Gauge}(C, n) \circ (i_2, k))$ meets $\text{LowerArc}(C)$.
- (50) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \text{len Gauge}(C, n + 1)$ and $1 < i_2$ and $i_2 < \text{len Gauge}(C, n + 1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n + 1)$ and $\text{Gauge}(C, n + 1) \circ (i_1, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$ and $\text{Gauge}(C, n + 1) \circ (i_2, j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$. Then $\mathcal{L}(\text{Gauge}(C, n + 1) \circ (i_2, j), \text{Gauge}(C, n + 1) \circ (i_2, k)) \cup \mathcal{L}(\text{Gauge}(C, n + 1) \circ (i_2, k), \text{Gauge}(C, n + 1) \circ (i_1, k))$ meets $\text{UpperArc}(C)$.
- (51) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \text{len Gauge}(C, n + 1)$ and $1 < i_2$ and $i_2 < \text{len Gauge}(C, n + 1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n + 1)$ and $\text{Gauge}(C, n + 1) \circ (i_1, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$ and $\text{Gauge}(C, n + 1) \circ (i_2, j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$. Then $\mathcal{L}(\text{Gauge}(C, n + 1) \circ (i_2, j), \text{Gauge}(C, n + 1) \circ (i_2, k)) \cup \mathcal{L}(\text{Gauge}(C, n + 1) \circ (i_2, k), \text{Gauge}(C, n + 1) \circ (i_1, k))$ meets $\text{LowerArc}(C)$.
- (52) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n + 1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n + 1)$ and $\text{Gauge}(C, n + 1) \circ (i, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$ and $\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$. Then $\mathcal{L}(\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), j), \text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), k)) \cup \mathcal{L}(\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), k), \text{Gauge}(C, n + 1) \circ (i, k))$ meets $\text{UpperArc}(C)$.
- (53) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \text{len Gauge}(C, n + 1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \text{width Gauge}(C, n + 1)$ and $\text{Gauge}(C, n + 1) \circ (i, k) \in \text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$ and $\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), j) \in \text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n + 1)))$. Then $\mathcal{L}(\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), j), \text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), k)) \cup \mathcal{L}(\text{Gauge}(C, n + 1) \circ (\text{Center Gauge}(C, n + 1), k), \text{Gauge}(C, n + 1) \circ (i, k))$ meets $\text{LowerArc}(C)$.

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