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Some Properties of Cells and Gauges[1]

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The articles [24], [27], [2], [12], [26], [25], [1], [4], [13], [28], [17], [6], [23], [3], [22], [9], [10], [5], [14], [11], [20], [7], [21], [19], [8], [15], [18], and [16] provide the notation and terminology for this paper.

We use the following convention: C is a simple closed curve, i, j, n are natural numbers, and p is a point of $\mathcal{E}^2_{\rm T}$.

We now state a number of propositions:

(2)[1] If $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $\langle i+1, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$, then $\rho(\mathrm{Gauge}(C,n)\circ(1,1), \mathrm{Gauge}(C,n)\circ(2,1)) = (\mathrm{Gauge}(C,n)\circ(i+1,j))_{\mathbf{1}} - (\mathrm{Gauge}(C,n)\circ(i,j))_{\mathbf{1}}$.

(3) If $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $\langle i, j+1\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$, then $\rho(\mathrm{Gauge}(C,n)\circ(1,1), \mathrm{Gauge}(C,n)\circ(1,2)) = (\mathrm{Gauge}(C,n)\circ(i,j+1))_{\mathbf{2}} - (\mathrm{Gauge}(C,n)\circ(i,j))_{\mathbf{2}}$.

(4) For every subset S of $\mathcal{E}^2_{\rm T}$ such that S is Bounded holds $\mathrm{proj1}^\circ S$ is bounded.

(5) Let C_1 be a non empty compact subset of $\mathcal{E}^2_{\rm T}$ and C_2, S be non empty subsets of $\mathcal{E}^2_{\rm T}$. If $S = C_1 \cup C_2$ and $\mathrm{proj1}^\circ C_2$ is non empty and lower bounded, then $\text{W-bound}(S) = \min(\text{W-bound}(C_1), \text{W-bound}(C_2))$.

(6) For every subset X of $\mathcal{E}^2_{\rm T}$ such that $p \in X$ and X is Bounded holds $\text{W-bound}(X) \leq p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leq \text{E-bound}(X)$ and $\text{S-bound}(X) \leq p_{\mathbf{2}}$ and $p_{\mathbf{2}} \leq \text{N-bound}(X)$.

(7) $p \in \mathrm{WestHalfline}\, p$ and $p \in \mathrm{EastHalfline}\, p$ and $p \in \mathrm{NorthHalfline}\, p$ and $p \in \mathrm{SouthHalfline}\, p$.

(8) $\mathrm{WestHalfline}\, p$ is non Bounded.

(9) $\mathrm{EastHalfline}\, p$ is non Bounded.

(10) $\mathrm{NorthHalfline}\, p$ is non Bounded.

(11) $\mathrm{SouthHalfline}\, p$ is non Bounded.

Let C be a compact subset of $\mathcal{E}^2_{\rm T}$. One can verify that $\mathrm{UBD}\,C$ is non empty.

The following propositions are true:

(12) For every compact subset C of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{UBD}\,C$ is a component of $C^{\rm c}$.

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[1] The proposition (1) has been removed.

(13) Let C be a compact subset of $\mathcal{E}^2_{\rm T}$ and W_1 be a connected subset of $\mathcal{E}^2_{\rm T}$. If W_1 is non Bounded and W_1 misses C, then $W_1 \subseteq \mathrm{UBD}\, C$.

(14) For every compact subset C of $\mathcal{E}^2_{\rm T}$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{WestHalfline}\, p$ misses C holds $\mathrm{WestHalfline}\, p \subseteq \mathrm{UBD}\, C$.

(15) For every compact subset C of $\mathcal{E}^2_{\rm T}$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{EastHalfline}\, p$ misses C holds $\mathrm{EastHalfline}\, p \subseteq \mathrm{UBD}\, C$.

(16) For every compact subset C of $\mathcal{E}^2_{\rm T}$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{SouthHalfline}\, p$ misses C holds $\mathrm{SouthHalfline}\, p \subseteq \mathrm{UBD}\, C$.

(17) For every compact subset C of $\mathcal{E}^2_{\rm T}$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{NorthHalfline}\, p$ misses C holds $\mathrm{NorthHalfline}\, p \subseteq \mathrm{UBD}\, C$.

(18) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{BDD}\, C \neq \emptyset$ holds $\text{W-bound}(C) \leq \text{W-bound}(\mathrm{BDD}\, C)$.

(19) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{BDD}\, C \neq \emptyset$ holds $\text{E-bound}(C) \geq \text{E-bound}(\mathrm{BDD}\, C)$.

(20) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{BDD}\, C \neq \emptyset$ holds $\text{S-bound}(C) \leq \text{S-bound}(\mathrm{BDD}\, C)$.

(21) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $\mathrm{BDD}\, C \neq \emptyset$ holds $\text{N-bound}(C) \geq \text{N-bound}(\mathrm{BDD}\, C)$.

(22) Let C be a compact non vertical subset of $\mathcal{E}^2_{\rm T}$ and I be an integer. If $p \in \mathrm{BDD}\, C$ and $I = \lfloor \frac{p_{\mathbf{1}} - \text{W-bound}(C)}{\text{E-bound}(C) - \text{W-bound}(C)} \cdot 2^n + 2 \rfloor$, then $1 < I$.

(23) Let C be a compact non vertical subset of $\mathcal{E}^2_{\rm T}$ and I be an integer. If $p \in \mathrm{BDD}\, C$ and $I = \lfloor \frac{p_{\mathbf{1}} - \text{W-bound}(C)}{\text{E-bound}(C) - \text{W-bound}(C)} \cdot 2^n + 2 \rfloor$, then $I + 1 \leq \mathrm{len}\, \mathrm{Gauge}(C, n)$.

(24) Let C be a compact non horizontal subset of $\mathcal{E}^2_{\rm T}$ and J be an integer. If $p \in \mathrm{BDD}\, C$ and $J = \lfloor \frac{p_{\mathbf{2}} - \text{S-bound}(C)}{\text{N-bound}(C) - \text{S-bound}(C)} \cdot 2^n + 2 \rfloor$, then $1 < J$ and $J + 1 \leq \mathrm{width}\, \mathrm{Gauge}(C, n)$.

(25) For every integer I such that $I = \lfloor \frac{p_{\mathbf{1}} - \text{W-bound}(C)}{\text{E-bound}(C) - \text{W-bound}(C)} \cdot 2^n + 2 \rfloor$ holds $\text{W-bound}(C) + \frac{\text{E-bound}(C) - \text{W-bound}(C)}{2^n} \cdot (I - 2) \leq p_{\mathbf{1}}$.

(26) For every integer I such that $I = \lfloor \frac{p_{\mathbf{1}} - \text{W-bound}(C)}{\text{E-bound}(C) - \text{W-bound}(C)} \cdot 2^n + 2 \rfloor$ holds $p_{\mathbf{1}} < \text{W-bound}(C) + \frac{\text{E-bound}(C) - \text{W-bound}(C)}{2^n} \cdot (I - 1)$.

(27) For every integer J such that $J = \lfloor \frac{p_{\mathbf{2}} - \text{S-bound}(C)}{\text{N-bound}(C) - \text{S-bound}(C)} \cdot 2^n + 2 \rfloor$ holds $\text{S-bound}(C) + \frac{\text{N-bound}(C) - \text{S-bound}(C)}{2^n} \cdot (J - 2) \leq p_{\mathbf{2}}$.

(28) For every integer J such that $J = \lfloor \frac{p_{\mathbf{2}} - \text{S-bound}(C)}{\text{N-bound}(C) - \text{S-bound}(C)} \cdot 2^n + 2 \rfloor$ holds $p_{\mathbf{2}} < \text{S-bound}(C) + \frac{\text{N-bound}(C) - \text{S-bound}(C)}{2^n} \cdot (J - 1)$.

(29) Let C be a closed subset of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2$. If $p \in \mathrm{BDD}\, C$, then there exists a real number r such that $r > 0$ and $\mathrm{Ball}(p, r) \subseteq \mathrm{BDD}\, C$.

(30) Let p, q be points of $\mathcal{E}^2_{\rm T}$ and r be a real number. Suppose $\rho(\mathrm{Gauge}(C, n) \circ (1, 1), \mathrm{Gauge}(C, n) \circ (1, 2)) < r$ and $\rho(\mathrm{Gauge}(C, n) \circ (1, 1), \mathrm{Gauge}(C, n) \circ (2, 1)) < r$ and $p \in \mathrm{cell}(\mathrm{Gauge}(C, n), i, j)$ and $q \in \mathrm{cell}(\mathrm{Gauge}(C, n), i, j)$ and $1 \leq i$ and $i + 1 \leq \mathrm{len}\, \mathrm{Gauge}(C, n)$ and $1 \leq j$ and $j + 1 \leq \mathrm{width}\, \mathrm{Gauge}(C, n)$. Then $\rho(p, q) < 2 \cdot r$.

(31) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \mathrm{BDD}\, C$ holds $p_{\mathbf{2}} \neq \text{N-bound}(\mathrm{BDD}\, C)$.

(32) For every compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in \mathrm{BDD}\,C$ holds $p_{\mathbf{1}} \neq \text{E-bound}(\mathrm{BDD}\,C)$.

(33) For every compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in \mathrm{BDD}\,C$ holds $p_{\mathbf{2}} \neq \text{S-bound}(\mathrm{BDD}\,C)$.

(34) For every compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in \mathrm{BDD}\,C$ holds $p_{\mathbf{1}} \neq \text{W-bound}(\mathrm{BDD}\,C)$.

(35) Suppose $p \in \mathrm{BDD}\,C$. Then there exist natural numbers n, i, j such that $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 < j$ and $j < \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $p_{\mathbf{1}} \neq (\mathrm{Gauge}(C,n) \circ (i,j))_{\mathbf{1}}$ and $p \in \mathrm{cell}(\mathrm{Gauge}(C,n),i,j)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$.

(36) For every subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is Bounded holds $\mathrm{UBD}\,C$ is non empty.

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