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Upper and Lower Sequence on the Cage, Upper and Lower Arcs[1]

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The articles [25], [30], [2], [4], [3], [29], [5], [14], [27], [20], [24], [13], [1], [23], [10], [11], [8], [28], [16], [12], [21], [26], [7], [18], [19], [6], [22], [9], [15], and [17] provide the notation and terminology for this paper.

In this paper n is a natural number.

We now state two propositions:

(1) Let G be a Go-board and i_1, i_2, j_1, j_2 be natural numbers. Suppose $1 \leq j_1$ and $j_1 \leq \mathrm{width}\, G$ and $1 \leq j_2$ and $j_2 \leq \mathrm{width}\, G$ and $1 \leq i_1$ and $i_1 < i_2$ and $i_2 \leq \mathrm{len}\, G$. Then $(G \circ (i_1, j_1))_{\mathbf{1}} < (G \circ (i_2, j_2))_{\mathbf{1}}$.

(2) Let G be a Go-board and i_1, i_2, j_1, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \mathrm{len}\, G$ and $1 \leq i_2$ and $i_2 \leq \mathrm{len}\, G$ and $1 \leq j_1$ and $j_1 < j_2$ and $j_2 \leq \mathrm{width}\, G$. Then $(G \circ (i_1, j_1))_{\mathbf{2}} < (G \circ (i_2, j_2))_{\mathbf{2}}$.

Let f be a non empty finite sequence and let g be a finite sequence. Observe that $f \frown g$ is non empty.

We now state a number of propositions:

(3) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n) -: \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))) \cap \widetilde{\mathcal{L}}(\mathrm{Cage}(C,n) :- \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))) = \{\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))), \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))\}$.

(4) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{UpperSeq}(C,n) = (\mathrm{Cage}(C,n) \circlearrowleft \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))) :- \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(5) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng}\, \mathrm{UpperSeq}(C,n)$ and $\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

(6) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{W}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng}\, \mathrm{UpperSeq}(C,n)$ and $\mathrm{W}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

(7) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng}\, \mathrm{UpperSeq}(C,n)$ and $\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

(8) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng}\, \mathrm{UpperSeq}(C,n)$ and $\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

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(9) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{E_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,UpperSeq}(C,n)$ and $\mathrm{E_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

(10) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{E_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,LowerSeq}(C,n)$ and $\mathrm{E_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$.

(11) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{E_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,LowerSeq}(C,n)$ and $\mathrm{E_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$.

(12) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{S_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,LowerSeq}(C,n)$ and $\mathrm{S_{max}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$.

(13) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{S_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,LowerSeq}(C,n)$ and $\mathrm{S_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$.

(14) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{W_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \mathrm{rng\,LowerSeq}(C,n)$ and $\mathrm{W_{min}}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$.

(15) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{N_{min}}(Y) \in X$ holds $\mathrm{N_{min}}(X) = \mathrm{N_{min}}(Y)$.

(16) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{N_{max}}(Y) \in X$ holds $\mathrm{N_{max}}(X) = \mathrm{N_{max}}(Y)$.

(17) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{E_{min}}(Y) \in X$ holds $\mathrm{E_{min}}(X) = \mathrm{E_{min}}(Y)$.

(18) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{E_{max}}(Y) \in X$ holds $\mathrm{E_{max}}(X) = \mathrm{E_{max}}(Y)$.

(19) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{S_{min}}(Y) \in X$ holds $\mathrm{S_{min}}(X) = \mathrm{S_{min}}(Y)$.

(20) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{S_{max}}(Y) \in X$ holds $\mathrm{S_{max}}(X) = \mathrm{S_{max}}(Y)$.

(21) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{W_{min}}(Y) \in X$ holds $\mathrm{W_{min}}(X) = \mathrm{W_{min}}(Y)$.

(22) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $X \subseteq Y$ and $\mathrm{W_{max}}(Y) \in X$ holds $\mathrm{W_{max}}(X) = \mathrm{W_{max}}(Y)$.

(23) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{N\text{-}bound}(X) < \mathrm{N\text{-}bound}(Y)$ holds $\mathrm{N\text{-}bound}(X \cup Y) = \mathrm{N\text{-}bound}(Y)$.

(24) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{E\text{-}bound}(X) < \mathrm{E\text{-}bound}(Y)$ holds $\mathrm{E\text{-}bound}(X \cup Y) = \mathrm{E\text{-}bound}(Y)$.

(25) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{S\text{-}bound}(X) < \mathrm{S\text{-}bound}(Y)$ holds $\mathrm{S\text{-}bound}(X \cup Y) = \mathrm{S\text{-}bound}(X)$.

(26) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{W\text{-}bound}(X) < \mathrm{W\text{-}bound}(Y)$ holds $\mathrm{W\text{-}bound}(X \cup Y) = \mathrm{W\text{-}bound}(X)$.

(27) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{N\text{-}bound}(X) < \mathrm{N\text{-}bound}(Y)$ holds $\mathrm{N_{min}}(X \cup Y) = \mathrm{N_{min}}(Y)$.

(28) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{N\text{-}bound}(X) < \mathrm{N\text{-}bound}(Y)$ holds $\mathrm{N_{max}}(X \cup Y) = \mathrm{N_{max}}(Y)$.

(29) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\mathrm{T}}$ such that $\mathrm{E\text{-}bound}(X) < \mathrm{E\text{-}bound}(Y)$ holds $\mathrm{E_{min}}(X \cup Y) = \mathrm{E_{min}}(Y)$.

(30) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\rm T}$ such that E-bound$(X) <$ E-bound(Y) holds $\mathrm{E}_{\max}(X \cup Y) = \mathrm{E}_{\max}(Y)$.

(31) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\rm T}$ such that S-bound$(X) <$ S-bound(Y) holds $\mathrm{S}_{\min}(X \cup Y) = \mathrm{S}_{\min}(X)$.

(32) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\rm T}$ such that S-bound$(X) <$ S-bound(Y) holds $\mathrm{S}_{\max}(X \cup Y) = \mathrm{S}_{\max}(X)$.

(33) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\rm T}$ such that W-bound$(X) <$ W-bound(Y) holds $\mathrm{W}_{\min}(X \cup Y) = \mathrm{W}_{\min}(X)$.

(34) For all non empty compact subsets X, Y of $\mathcal{E}^2_{\rm T}$ such that W-bound$(X) <$ W-bound(Y) holds $\mathrm{W}_{\max}(X \cup Y) = \mathrm{W}_{\max}(X)$.

(35) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If f is a special sequence and $p \in \widetilde{\mathcal{L}}(f)$, then $(\downharpoonleft p, f)_{\mathrm{len} \downharpoonleft p, f} = f_{\mathrm{len}\, f}$.

(36) Let f be a non constant standard special circular sequence, p, q be points of $\mathcal{E}^2_{\rm T}$, and g be a connected subset of $\mathcal{E}^2_{\rm T}$. If $p \in \mathrm{RightComp}(f)$ and $q \in \mathrm{LeftComp}(f)$ and $p \in g$ and $q \in g$, then g meets $\widetilde{\mathcal{L}}(f)$.

Let us note that there exists special sequence finite sequence of elements of $\mathcal{E}^2_{\rm T}$ which is non constant, standard, and s.c.c..

One can prove the following propositions:

(37) For every S-sequence f in $\mathbb{R}^2$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in \mathrm{rng}\, f$ holds $\downharpoonleft p, f = \mathrm{mid}(f, p \looparrowleft f, \mathrm{len}\, f)$.

(38) Let M be a Go-board and f be a S-sequence in $\mathbb{R}^2$. Suppose f is a sequence which elements belong to M. Let p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \mathrm{rng}\, f$, then $\downharpoonright f, p$ is a sequence which elements belong to M.

(39) Let M be a Go-board and f be a S-sequence in $\mathbb{R}^2$. Suppose f is a sequence which elements belong to M. Let p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \mathrm{rng}\, f$, then $\downharpoonleft p, f$ is a sequence which elements belong to M.

(40) Let G be a Go-board and f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose f is a sequence which elements belong to G. Let i, j be natural numbers. If $1 \leq i$ and $i \leq \mathrm{len}\, G$ and $1 \leq j$ and $j \leq \mathrm{width}\, G$, then if $G \circ (i, j) \in \widetilde{\mathcal{L}}(f)$, then $G \circ (i, j) \in \mathrm{rng}\, f$.

(41) Let f be a S-sequence in $\mathbb{R}^2$ and g be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) g is unfolded, s.n.c., and one-to-one,

(ii) $\widetilde{\mathcal{L}}(f) \cap \widetilde{\mathcal{L}}(g) = \{f_1\}$,

(iii) $f_1 = g_{\mathrm{len}\, g}$,

(iv) for every natural number i such that $1 \leq i$ and $i + 2 \leq \mathrm{len}\, f$ holds $\mathcal{L}(f, i) \cap \mathcal{L}(f_{\mathrm{len}\, f}, g_1) = \emptyset$, and

(v) for every natural number i such that $2 \leq i$ and $i + 1 \leq \mathrm{len}\, g$ holds $\mathcal{L}(g, i) \cap \mathcal{L}(f_{\mathrm{len}\, f}, g_1) = \emptyset$.

Then $f \frown g$ is s.c.c..

(42) Let C be a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\rm T}$. Then there exists a natural number i such that $1 \leq i$ and $i + 1 \leq \mathrm{len}\, \mathrm{Gauge}(C, n)$ and $\mathrm{W}_{\min}(C) \in \mathrm{cell}(\mathrm{Gauge}(C, n), 1, i)$ and $\mathrm{W}_{\min}(C) \neq \mathrm{Gauge}(C, n) \circ (2, i)$.

(43) For every S-sequence f in $\mathbb{R}^2$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in \widetilde{\mathcal{L}}(f)$ and $f(\mathrm{len}\, f) \in \widetilde{\mathcal{L}}(\downharpoonright f, p)$ holds $f(\mathrm{len}\, f) = p$.

(44) For every non empty finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ and for every point p of $\mathcal{E}^2_{\rm T}$ holds $\downharpoonright f, p \neq \emptyset$.

(45) For every S-sequence f in $\mathbb{R}^2$ and for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in \widetilde{\mathcal{L}}(f)$ holds $(\downharpoonleft f, p)_{{\rm len} \downharpoonleft f,p} = p$.

(46) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \widetilde{\mathcal{L}}({\rm UpperSeq}(C,n))$ and $p_{\bf 1} = \text{E-bound}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))$, then $p = {\rm E}_{\rm max}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))$.

(47) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \widetilde{\mathcal{L}}({\rm LowerSeq}(C,n))$ and $p_{\bf 1} = \text{W-bound}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))$, then $p = {\rm W}_{\rm min}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))$.

(48) Let G be a Go-board, f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$, and k be a natural number. Suppose $1 \leq k$ and $k < {\rm len}\, f$ and $f \frown g$ is a sequence which elements belong to G. Then ${\rm left\_cell}(f \frown g, k, G) = {\rm left\_cell}(f, k, G)$ and ${\rm right\_cell}(f \frown g, k, G) = {\rm right\_cell}(f, k, G)$.

(49) Let D be a set, f, g be finite sequences of elements of D, and i be a natural number. If $i \leq {\rm len}\, f$, then $(f \frown\frown g) \restriction i = f \restriction i$.

(50) For every set D and for all finite sequences f, g of elements of D holds $(f \frown\frown g) \restriction {\rm len}\, f = f$.

(51) Let G be a Go-board, f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$, and k be a natural number. Suppose $1 \leq k$ and $k < {\rm len}\, f$ and $f \frown\frown g$ is a sequence which elements belong to G. Then ${\rm left\_cell}(f \frown\frown g, k, G) = {\rm left\_cell}(f, k, G)$ and ${\rm right\_cell}(f \frown\frown g, k, G) = {\rm right\_cell}(f, k, G)$.

(52) Let G be a Go-board, f be a S-sequence in $\mathbb{R}^2$, p be a point of $\mathcal{E}^2_{\rm T}$, and k be a natural number. Suppose $1 \leq k$ and $k < p \leftarrow\!\!\!\!\!\lrcorner\, f$ and f is a sequence which elements belong to G and $p \in {\rm rng}\, f$. Then ${\rm left\_cell}(\downharpoonleft f, p, k, G) = {\rm left\_cell}(f, k, G)$ and ${\rm right\_cell}(\downharpoonleft f, p, k, G) = {\rm right\_cell}(f, k, G)$.

(53) Let G be a Go-board, f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$, p be a point of $\mathcal{E}^2_{\rm T}$, and k be a natural number. Suppose $1 \leq k$ and $k < p \leftarrow\!\!\!\!\!\lrcorner\, f$ and f is a sequence which elements belong to G. Then ${\rm left\_cell}(f -: p, k, G) = {\rm left\_cell}(f, k, G)$ and ${\rm right\_cell}(f -: p, k, G) = {\rm right\_cell}(f, k, G)$.

(54) Let f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) f is unfolded, s.n.c., and one-to-one,

(ii) g is unfolded, s.n.c., and one-to-one,

(iii) $f_{{\rm len}\, f} = g_1$, and

(iv) $\widetilde{\mathcal{L}}(f) \cap \widetilde{\mathcal{L}}(g) = \{g_1\}$.

Then $f \frown\frown g$ is s.n.c..

(55) Let f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose f is one-to-one and g is one-to-one and ${\rm rng}\, f \cap {\rm rng}\, g \subseteq \{g_1\}$. Then $f \frown\frown g$ is one-to-one.

(56) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If f is a special sequence and $p \in {\rm rng}\, f$ and $p \neq f(1)$, then ${\rm Index}(p, f) + 1 = p \leftarrow\!\!\!\!\!\lrcorner\, f$.

(57) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$ and i, j, k be natural numbers. Suppose $1 < i$ and $i < {\rm len}\,{\rm Gauge}(C,n)$ and $1 \leq j$ and $k \leq {\rm width}\,{\rm Gauge}(C,n)$ and ${\rm Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}({\rm UpperSeq}(C,n))$ and ${\rm Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}({\rm LowerSeq}(C,n))$. Then $j \neq k$.

(58) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < {\rm len}\,{\rm Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq {\rm width}\,{\rm Gauge}(C,n)$ and $\mathcal{L}({\rm Gauge}(C,n) \circ (i,j), {\rm Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}({\rm UpperSeq}(C,n)) = \{{\rm Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}({\rm Gauge}(C,n) \circ (i,j), {\rm Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}({\rm LowerSeq}(C,n)) = \{{\rm Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}({\rm Gauge}(C,n) \circ (i,j), {\rm Gauge}(C,n) \circ (i,k))$ meets ${\rm LowerArc}(C)$.

(59) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width} \operatorname{Gauge}(C,n)$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}(\operatorname{UpperSeq}(C,n)) = \{\operatorname{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}(\operatorname{LowerSeq}(C,n)) = \{\operatorname{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k))$ meets $\operatorname{UpperArc}(C)$.

(60) Let C be a simple closed curve and i, j, k be natural numbers. Suppose that $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width} \operatorname{Gauge}(C,n)$ and $n > 0$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \{\operatorname{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \{\operatorname{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k))$ meets $\operatorname{LowerArc}(C)$.

(61) Let C be a simple closed curve and i, j, k be natural numbers. Suppose that $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width} \operatorname{Gauge}(C,n)$ and $n > 0$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \{\operatorname{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k)) \cap \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \{\operatorname{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,k))$ meets $\operatorname{UpperArc}(C)$.

(62) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and j be a natural number. Suppose $\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1), j) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$ and $1 \leq j$ and $j \leq \operatorname{width} \operatorname{Gauge}(C,n+1)$. Then $\mathcal{L}(\operatorname{Gauge}(C,1) \circ (\operatorname{Center} \operatorname{Gauge}(C,1),1), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j))$ meets $\operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$.

(63) Let C be a simple closed curve and j, k be natural numbers. Suppose that

(i) $1 \leq j$,

(ii) $j \leq k$,

(iii) $k \leq \operatorname{width} \operatorname{Gauge}(C,n+1)$,

(iv) $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)) \cap \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1))) = \{\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)\}$, and

(v) $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)) \cap \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1))) = \{\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j)\}$.

Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k))$ meets $\operatorname{LowerArc}(C)$.

(64) Let C be a simple closed curve and j, k be natural numbers. Suppose that

(i) $1 \leq j$,

(ii) $j \leq k$,

(iii) $k \leq \operatorname{width} \operatorname{Gauge}(C,n+1)$,

(iv) $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)) \cap \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1))) = \{\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)\}$, and

(v) $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k)) \cap \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1))) = \{\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j)\}$.

Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center} \operatorname{Gauge}(C,n+1),k))$ meets $\operatorname{UpperArc}(C)$.

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