

# Linear Combinations in Left Module over Associative Ring<sup>1</sup>

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**Summary.** Notion of linear combination of vectors in Left Module over Associative Ring, defined as function from the carrier of Left Module over Associative Ring to the carrier of this Ring. The following operations are included: addition, subtraction of combinations and multiplication of a combination by a scalar of the Ring. Following it, the sum of a finite set of vectors and the sum of linear combination is defined. Many theorems are proved. This article originated as a generalization of the article [8].

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The articles [6], [1], [10], [2], [3], [7], [4], [5], [9], and [8] provide the notation and terminology for this paper.

In this paper  $R$  is a ring and  $V$  is a left module over  $R$ .

Let us consider  $R$  and let us consider  $V$ . We introduce submodule of  $V$  as a synonym of subspace of  $V$ .

In the sequel  $R$  denotes an integral domain,  $V$  denotes a left module over  $R$ ,  $L$  denotes a linear combination of  $V$ , and  $a$  denotes a scalar of  $R$ .

We now state two propositions:

(59)<sup>1</sup> If  $a \neq 0_R$ , then the support of  $a \cdot L =$  the support of  $L$ .

(78)<sup>2</sup>  $\Sigma(a \cdot L) = a \cdot \Sigma L$ .

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<sup>1</sup>The propositions (1)–(58) have been removed.

<sup>2</sup>The propositions (60)–(77) have been removed.

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