JOURNAL OF FORMALIZED MATHEMATICS
Volume 1, Released 1989, Published 2003
Inst. of Computer Science, Univ. of Białystok

Tuples, Projections and Cartesian Products

Andrzej Trybulec
Warsaw University
Białystok

Summary. The purpose of this article is to define projections of ordered pairs, and to introduce triples and quadruples, and their projections. The theorems in this paper may be roughly divided into two groups: theorems describing basic properties of introduced concepts and theorems related to the regularity, analogous to those proved for ordered pairs by Cz. Byliński [1]. Cartesian products of subsets are redefined as subsets of Cartesian products.

MML Identifier: MCART_1.

WWW: http://mizar.org/JFM/Vol1/mcart_1.html

The articles [3], [2], [1], and [4] provide the notation and terminology for this paper.

We adopt the following rules: v, x, x_1, x_2, x_3, x_4, y, y_1, y_2, y_3, y_4, z denote sets and X, X_1, X_2, X_3, X_4, Y, Y_1, Y_2, Y_3, Y_4, Y_5, Z denote sets.

The following propositions are true:

(1) If $X \neq \emptyset$, then there exists Y such that $Y \in X$ and Y misses X.

(2) If $X \neq \emptyset$, then there exists Y such that $Y \in X$ and for every Y_1 such that $Y_1 \in Y$ holds Y_1 misses X.

(3) If $X \neq \emptyset$, then there exists Y such that $Y \in X$ and for all Y_1, Y_2 such that $Y_1 \in Y_2$ and $Y_2 \in Y$ holds Y_1 misses X.

(4) If $X \neq \emptyset$, then there exists Y such that $Y \in X$ and for all Y_1, Y_2, Y_3 such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y$ holds Y_1 misses X.

(5) If $X \neq \emptyset$, then there exists Y such that $Y \in X$ and for all Y_1, Y_2, Y_3, Y_4 such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y$ holds Y_1 misses X.

(6) Suppose $X \neq \emptyset$. Then there exists Y such that $Y \in X$ and for all Y_1, Y_2, Y_3, Y_4, Y_5 such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y_5$ and $Y_5 \in Y$ holds Y_1 misses X.

Let us consider x. Let us assume that there exist sets x_1, x_2 such that $x = \langle x_1, x_2 \rangle$. The functor $x_{\mathbf{1}}$ is defined by:

(Def. 1) If $x = \langle y_1, y_2 \rangle$, then $x_{\mathbf{1}} = y_1$.

The functor $x_{\mathbf{2}}$ is defined by:

(Def. 2) If $x = \langle y_1, y_2 \rangle$, then $x_{\mathbf{2}} = y_2$.

We now state a number of propositions:

(7) $\langle x, y \rangle_{\mathbf{1}} = x$ and $\langle x, y \rangle_{\mathbf{2}} = y$.

(8) If there exist x, y such that $z = \langle x, y\rangle$, then $\langle z_{\mathbf{1}}, z_{\mathbf{2}}\rangle = z$.

(9) If $X \neq \emptyset$, then there exists v such that $v \in X$ and it is not true that there exist x, y such that $x \in X$ or $y \in X$ but $v = \langle x, y\rangle$.

(10) If $z \in [: X, Y :]$, then $z_{\mathbf{1}} \in X$ and $z_{\mathbf{2}} \in Y$.

(11) If there exist x, y such that $z = \langle x, y\rangle$ and $z_{\mathbf{1}} \in X$ and $z_{\mathbf{2}} \in Y$, then $z \in [: X, Y :]$.

(12) If $z \in [: \{x\}, Y :]$, then $z_{\mathbf{1}} = x$ and $z_{\mathbf{2}} \in Y$.

(13) If $z \in [: X, \{y\} :]$, then $z_{\mathbf{1}} \in X$ and $z_{\mathbf{2}} = y$.

(14) If $z \in [: \{x\}, \{y\} :]$, then $z_{\mathbf{1}} = x$ and $z_{\mathbf{2}} = y$.

(15) If $z \in [: \{x_1, x_2\}, Y :]$, then $z_{\mathbf{1}} = x_1$ or $z_{\mathbf{1}} = x_2$ but $z_{\mathbf{2}} \in Y$.

(16) If $z \in [: X, \{y_1, y_2\} :]$, then $z_{\mathbf{1}} \in X$ but $z_{\mathbf{2}} = y_1$ or $z_{\mathbf{2}} = y_2$.

(17) If $z \in [: \{x_1, x_2\}, \{y\} :]$, then $z_{\mathbf{1}} = x_1$ or $z_{\mathbf{1}} = x_2$ but $z_{\mathbf{2}} = y$.

(18) If $z \in [: \{x\}, \{y_1, y_2\} :]$, then $z_{\mathbf{1}} = x$ but $z_{\mathbf{2}} = y_1$ or $z_{\mathbf{2}} = y_2$.

(19) If $z \in [: \{x_1, x_2\}, \{y_1, y_2\} :]$, then $z_{\mathbf{1}} = x_1$ or $z_{\mathbf{1}} = x_2$ but $z_{\mathbf{2}} = y_1$ or $z_{\mathbf{2}} = y_2$.

(20) If there exist y, z such that $x = \langle y, z\rangle$, then $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$.

(23)[1] If $x \in [: X, Y :]$, then $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}\rangle$.

(24) If $X \neq \emptyset$ and $Y \neq \emptyset$, then for every element x of $[: X, Y :]$ holds $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}\rangle$.

(25) $[: \{x_1, x_2\}, \{y_1, y_2\} :] = \{\langle x_1, y_1\rangle, \langle x_1, y_2\rangle, \langle x_2, y_1\rangle, \langle x_2, y_2\rangle\}$.

(26) If $X \neq \emptyset$ and $Y \neq \emptyset$, then for every element x of $[: X, Y :]$ holds $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$.

Let us consider x_1, x_2, x_3. The functor $\langle x_1, x_2, x_3\rangle$ is defined by:

(Def. 3) $\langle x_1, x_2, x_3\rangle = \langle\langle x_1, x_2\rangle, x_3\rangle$.

Next we state two propositions:

(28)[2] If $\langle x_1, x_2, x_3\rangle = \langle y_1, y_2, y_3\rangle$, then $x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$.

(29) If $X \neq \emptyset$, then there exists v such that $v \in X$ and it is not true that there exist x, y, z such that $x \in X$ or $y \in X$ but $v = \langle x, y, z\rangle$.

Let us consider x_1, x_2, x_3, x_4. The functor $\langle x_1, x_2, x_3, x_4\rangle$ is defined by:

(Def. 4) $\langle x_1, x_2, x_3, x_4\rangle = \langle\langle x_1, x_2, x_3\rangle, x_4\rangle$.

Next we state several propositions:

(31)[3] $\langle x_1, x_2, x_3, x_4\rangle = \langle\langle\langle x_1, x_2\rangle, x_3\rangle, x_4\rangle$.

(32) $\langle x_1, x_2, x_3, x_4\rangle = \langle\langle x_1, x_2\rangle, x_3, x_4\rangle$.

(33) If $\langle x_1, x_2, x_3, x_4\rangle = \langle y_1, y_2, y_3, y_4\rangle$, then $x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$ and $x_4 = y_4$.

(34) If $X \neq \emptyset$, then there exists v such that $v \in X$ and it is not true that there exist x_1, x_2, x_3, x_4 such that $x_1 \in X$ or $x_2 \in X$ but $v = \langle x_1, x_2, x_3, x_4\rangle$.

(35) $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ iff $[: X_1, X_2, X_3 :] \neq \emptyset$.

[1] The propositions (21) and (22) have been removed.
[2] The proposition (27) has been removed.
[3] The proposition (30) has been removed.

In the sequel x_5 is an element of X_1, x_6 is an element of X_2, and x_7 is an element of X_3.

We now state a number of propositions:

(36) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$, then if $[:X_1, X_2, X_3:] = [:Y_1, Y_2, Y_3:]$, then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$.

(37) If $[:X_1, X_2, X_3:] \neq \emptyset$ and $[:X_1, X_2, X_3:] = [:Y_1, Y_2, Y_3:]$, then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$.

(38) If $[:X, X, X:] = [:Y, Y, Y:]$, then $X = Y$.

(39) $[:\{x_1\}, \{x_2\}, \{x_3\}:] = \{\langle x_1, x_2, x_3\rangle\}$.

(40) $[:\{x_1,y_1\}, \{x_2\}, \{x_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle y_1, x_2, x_3\rangle\}$.

(41) $[:\{x_1\}, \{x_2,y_2\}, \{x_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle x_1, y_2, x_3\rangle\}$.

(42) $[:\{x_1\}, \{x_2\}, \{x_3,y_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle x_1, x_2, y_3\rangle\}$.

(43) $[:\{x_1,y_1\}, \{x_2,y_2\}, \{x_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle y_1, x_2, x_3\rangle, \langle x_1, y_2, x_3\rangle, \langle y_1, y_2, x_3\rangle\}$.

(44) $[:\{x_1,y_1\}, \{x_2\}, \{x_3,y_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle y_1, x_2, x_3\rangle, \langle x_1, x_2, y_3\rangle, \langle y_1, x_2, y_3\rangle\}$.

(45) $[:\{x_1\}, \{x_2,y_2\}, \{x_3,y_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle x_1, y_2, x_3\rangle, \langle x_1, x_2, y_3\rangle, \langle x_1, y_2, y_3\rangle\}$.

(46) $[:\{x_1,y_1\}, \{x_2,y_2\}, \{x_3,y_3\}:] = \{\langle x_1, x_2, x_3\rangle, \langle x_1, y_2, x_3\rangle, \langle x_1, x_2, y_3\rangle, \langle x_1, y_2, y_3\rangle, \langle y_1, x_2, x_3\rangle, \langle y_1, y_2, x_3\rangle, \langle y_1, x_2, y_3\rangle, \langle y_1, y_2, y_3\rangle\}$.

Let us consider X_1, X_2, X_3. Let us assume that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$. Let x be an element of $[:X_1, X_2, X_3:]$. The functor $x_{\mathbf{1}}$ yields an element of X_1 and is defined as follows:

(Def. 5) If $x = \langle x_1, x_2, x_3\rangle$, then $x_{\mathbf{1}} = x_1$.

The functor $x_{\mathbf{2}}$ yields an element of X_2 and is defined as follows:

(Def. 6) If $x = \langle x_1, x_2, x_3\rangle$, then $x_{\mathbf{2}} = x_2$.

The functor $x_{\mathbf{3}}$ yields an element of X_3 and is defined by:

(Def. 7) If $x = \langle x_1, x_2, x_3\rangle$, then $x_{\mathbf{3}} = x_3$.

The following propositions are true:

(47) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$. Let x be an element of $[:X_1, X_2, X_3:]$ and given x_1, x_2, x_3. If $x = \langle x_1, x_2, x_3\rangle$, then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$.

(48) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$, then for every element x of $[:X_1, X_2, X_3:]$ holds $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}\rangle$.

(49) If $X \subseteq [:X, Y, Z:]$ or $X \subseteq [:Y, Z, X:]$ or $X \subseteq [:Z, X, Y:]$, then $X = \emptyset$.

(50) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$. Let x be an element of $[:X_1, X_2, X_3:]$. Then $x_{\mathbf{1}} = ((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}}$ and $x_{\mathbf{2}} = ((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{3}} = (x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{2}}$.

(51) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$, then for every element x of $[:X_1, X_2, X_3:]$ holds $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$ and $x \neq x_{\mathbf{3}}$.

(52) If $[:X_1, X_2, X_3:]$ meets $[:Y_1, Y_2, Y_3:]$, then X_1 meets Y_1 and X_2 meets Y_2 and X_3 meets Y_3.

(53) $[:X_1, X_2, X_3, X_4:] = [:[:[:X_1, X_2:], X_3:], X_4:]$.

(54) $[:[:X_1, X_2:], X_3, X_4:] = [:X_1, X_2, X_3, X_4:]$.

(55) $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ iff $[:X_1, X_2, X_3, X_4:] \neq \emptyset$.

(56) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$, then if $[:X_1, X_2, X_3, X_4:] = [:Y_1, Y_2, Y_3, Y_4:]$, then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$.

(57) If $[: X_1, X_2, X_3, X_4 :] \neq \emptyset$ and $[: X_1, X_2, X_3, X_4 :] = [: Y_1, Y_2, Y_3, Y_4 :]$, then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$.

(58) If $[: X, X, X, X :] = [: Y, Y, Y, Y :]$, then $X = Y$.

In the sequel x_8 denotes an element of X_4.

Let us consider X_1, X_2, X_3, X_4. Let us assume that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4 :]$. The functor $x_{\mathbf{1}}$ yielding an element of X_1 is defined as follows:

(Def. 8) If $x = \langle x_1, x_2, x_3, x_4 \rangle$, then $x_{\mathbf{1}} = x_1$.

The functor $x_{\mathbf{2}}$ yields an element of X_2 and is defined by:

(Def. 9) If $x = \langle x_1, x_2, x_3, x_4 \rangle$, then $x_{\mathbf{2}} = x_2$.

The functor $x_{\mathbf{3}}$ yields an element of X_3 and is defined as follows:

(Def. 10) If $x = \langle x_1, x_2, x_3, x_4 \rangle$, then $x_{\mathbf{3}} = x_3$.

The functor $x_{\mathbf{4}}$ yielding an element of X_4 is defined by:

(Def. 11) If $x = \langle x_1, x_2, x_3, x_4 \rangle$, then $x_{\mathbf{4}} = x_4$.

The following propositions are true:

(59) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4 :]$ and given x_1, x_2, x_3, x_4. If $x = \langle x_1, x_2, x_3, x_4 \rangle$, then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$.

(60) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$, then for every element x of $[: X_1, X_2, X_3, X_4 :]$ holds $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}, x_{\mathbf{4}} \rangle$.

(61) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4 :]$. Then $x_{\mathbf{1}} = (((x \textbf{ qua } \text{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}}$ and $x_{\mathbf{2}} = (((x \textbf{ qua } \text{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{3}} = ((x \textbf{ qua } \text{set})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{4}} = (x \textbf{ qua } \text{set})_{\mathbf{2}}$.

(62) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$, then for every element x of $[: X_1, X_2, X_3, X_4 :]$ holds $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$ and $x \neq x_{\mathbf{3}}$ and $x \neq x_{\mathbf{4}}$.

(63) If $X_1 \subseteq [: X_1, X_2, X_3, X_4 :]$ or $X_1 \subseteq [: X_2, X_3, X_4, X_1 :]$ or $X_1 \subseteq [: X_3, X_4, X_1, X_2 :]$ or $X_1 \subseteq [: X_4, X_1, X_2, X_3 :]$, then $X_1 = \emptyset$.

(64) If $[: X_1, X_2, X_3, X_4 :]$ meets $[: Y_1, Y_2, Y_3, Y_4 :]$, then X_1 meets Y_1 and X_2 meets Y_2 and X_3 meets Y_3 and X_4 meets Y_4.

(65) $[: \{x_1\}, \{x_2\}, \{x_3\}, \{x_4\} :] = \{\langle x_1, x_2, x_3, x_4 \rangle\}$.

(66) If $[: X, Y :] \neq \emptyset$, then for every element x of $[: X, Y :]$ holds $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$.

(67) If $x \in [: X, Y :]$, then $x \neq x_{\mathbf{1}}$ and $x \neq x_{\mathbf{2}}$.

For simplicity, we adopt the following convention: A_1 denotes a subset of X_1, A_2 denotes a subset of X_2, A_3 denotes a subset of X_3, A_4 denotes a subset of X_4, and x denotes an element of $[: X_1, X_2, X_3 :]$.

One can prove the following propositions:

(68) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$, then for all x_1, x_2, x_3 such that $x = \langle x_1, x_2, x_3 \rangle$ holds $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$.

(69) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and for all x_5, x_6, x_7 such that $x = \langle x_5, x_6, x_7 \rangle$ holds $y_1 = x_5$, then $y_1 = x_{\mathbf{1}}$.

(70) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and for all x_5, x_6, x_7 such that $x = \langle x_5, x_6, x_7 \rangle$ holds $y_2 = x_6$, then $y_2 = x_{\mathbf{2}}$.

(71) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and for all x_5, x_6, x_7 such that $x = \langle x_5, x_6, x_7 \rangle$ holds $y_3 = x_7$, then $y_3 = x_{\mathbf{3}}$.

(72) If $z \in [: X_1, X_2, X_3 :]$, then there exist x_1, x_2, x_3 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $z = \langle x_1, x_2, x_3 \rangle$.

(73) $\langle x_1, x_2, x_3 \rangle \in [: X_1, X_2, X_3 :]$ iff $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$.

(74) If for every z holds $z \in Z$ iff there exist x_1, x_2, x_3 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $z = \langle x_1, x_2, x_3 \rangle$, then $Z = [: X_1, X_2, X_3 :]$.

(75) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $Y_1 \neq \emptyset$ and $Y_2 \neq \emptyset$ and $Y_3 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3 :]$ and y be an element of $[: Y_1, Y_2, Y_3 :]$. If $x = y$, then $x_{\mathbf{1}} = y_{\mathbf{1}}$ and $x_{\mathbf{2}} = y_{\mathbf{2}}$ and $x_{\mathbf{3}} = y_{\mathbf{3}}$.

(76) For every element x of $[: X_1, X_2, X_3 :]$ such that $x \in [: A_1, A_2, A_3 :]$ holds $x_{\mathbf{1}} \in A_1$ and $x_{\mathbf{2}} \in A_2$ and $x_{\mathbf{3}} \in A_3$.

(77) If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$ and $X_3 \subseteq Y_3$, then $[: X_1, X_2, X_3 :] \subseteq [: Y_1, Y_2, Y_3 :]$.

In the sequel x denotes an element of $[: X_1, X_2, X_3, X_4 :]$.
The following propositions are true:

(78) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$, then for all x_1, x_2, x_3, x_4 such that $x = \langle x_1, x_2, x_3, x_4 \rangle$ holds $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$.

(79) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and for all x_5, x_6, x_7, x_8 such that $x = \langle x_5, x_6, x_7, x_8 \rangle$ holds $y_1 = x_5$, then $y_1 = x_{\mathbf{1}}$.

(80) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and for all x_5, x_6, x_7, x_8 such that $x = \langle x_5, x_6, x_7, x_8 \rangle$ holds $y_2 = x_6$, then $y_2 = x_{\mathbf{2}}$.

(81) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and for all x_5, x_6, x_7, x_8 such that $x = \langle x_5, x_6, x_7, x_8 \rangle$ holds $y_3 = x_7$, then $y_3 = x_{\mathbf{3}}$.

(82) If $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and for all x_5, x_6, x_7, x_8 such that $x = \langle x_5, x_6, x_7, x_8 \rangle$ holds $y_4 = x_8$, then $y_4 = x_{\mathbf{4}}$.

(83) If $z \in [: X_1, X_2, X_3, X_4 :]$, then there exist x_1, x_2, x_3, x_4 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $z = \langle x_1, x_2, x_3, x_4 \rangle$.

(84) $\langle x_1, x_2, x_3, x_4 \rangle \in [: X_1, X_2, X_3, X_4 :]$ iff $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$.

(85) If for every z holds $z \in Z$ iff there exist x_1, x_2, x_3, x_4 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $z = \langle x_1, x_2, x_3, x_4 \rangle$, then $Z = [: X_1, X_2, X_3, X_4 :]$.

(86) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $Y_1 \neq \emptyset$ and $Y_2 \neq \emptyset$ and $Y_3 \neq \emptyset$ and $Y_4 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4 :]$ and y be an element of $[: Y_1, Y_2, Y_3, Y_4 :]$. If $x = y$, then $x_{\mathbf{1}} = y_{\mathbf{1}}$ and $x_{\mathbf{2}} = y_{\mathbf{2}}$ and $x_{\mathbf{3}} = y_{\mathbf{3}}$ and $x_{\mathbf{4}} = y_{\mathbf{4}}$.

(87) For every element x of $[: X_1, X_2, X_3, X_4 :]$ such that $x \in [: A_1, A_2, A_3, A_4 :]$ holds $x_{\mathbf{1}} \in A_1$ and $x_{\mathbf{2}} \in A_2$ and $x_{\mathbf{3}} \in A_3$ and $x_{\mathbf{4}} \in A_4$.

(88) If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$ and $X_3 \subseteq Y_3$ and $X_4 \subseteq Y_4$, then $[: X_1, X_2, X_3, X_4 :] \subseteq [: Y_1, Y_2, Y_3, Y_4 :]$.

Let us consider X_1, X_2, A_1, A_2. Then $[: A_1, A_2 :]$ is a subset of $[: X_1, X_2 :]$.
Let us consider X_1, X_2, X_3, A_1, A_2, A_3. Then $[: A_1, A_2, A_3 :]$ is a subset of $[: X_1, X_2, X_3 :]$.
Let us consider X_1, X_2, X_3, X_4, A_1, A_2, A_3, A_4. Then $[: A_1, A_2, A_3, A_4 :]$ is a subset of $[: X_1, X_2, X_3, X_4 :]$.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[2] Andrzej Trybulec. Enumerated sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/enumset1.html.

[3] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[4] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

Received March 30, 1989

Published January 2, 2004