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Half Open Intervals in Real Numbers

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Summary. Left and right half open intervals in the real line are defined. Their properties are investigated. A class of all finite union of such intervals are, in a sense, closed by operations of union, intersection and the difference of sets.

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The articles [3], [6], [1], [4], [5], and [2] provide the notation and terminology for this paper.

In this paper s, g, h, r, p, p_1, p_2, q, q_1, q_2, x, y, z denote real numbers.

The following proposition is true

(2)[1] $y < x$ and $z < x$ iff $\max(y,z) < x$.

Let g, s be real numbers. The functor $[g,s[$ yields a subset of $\mathbb{R}$ and is defined by:

(Def. 1) $[g,s[= \{r; r$ ranges over real numbers: $g \leq r \wedge r < s\}$.

The functor $]g,s]$ yields a subset of $\mathbb{R}$ and is defined by:

(Def. 2) $]g,s] = \{r; r$ ranges over real numbers: $g < r \wedge r \leq s\}$.

We now state a number of propositions:

(3) $r \in [p,q[$ iff $p \leq r$ and $r < q$.

(4) $r \in]p,q]$ iff $p < r$ and $r \leq q$.

(5) For all g, s such that $g < s$ holds $[g,s[=]g,s[\cup \{g\}$.

(6) For all g, s such that $g < s$ holds $]g,s] =]g,s[\cup \{s\}$.

(7) $[g,g[= \emptyset$.

(8) $]g,g] = \emptyset$.

(9) If $p \leq g$, then $[g,p[= \emptyset$.

(10) If $p \leq g$, then $]g,p] = \emptyset$.

(11) If $g \leq p$ and $p \leq h$, then $[g,p[\cup [p,h[= [g,h[$.

(12) If $g \leq p$ and $p \leq h$, then $]g,p] \cup]p,h] =]g,h]$.

[1] The proposition (1) has been removed.

(13) If $g \leq p_1$ and $g \leq p_2$ and $p_1 \leq h$ and $p_2 \leq h$, then $[g,h] = [g,p_1[\cup [p_1,p_2] \cup]p_2,h]$.

(14) If $g < p_1$ and $g < p_2$ and $p_1 < h$ and $p_2 < h$, then $]g,h[=]g,p_1] \cup]p_1,p_2[\cup [p_2,h[$.

(15) $[q_1,q_2[\cap [p_1,p_2[= [\max(q_1,p_1),\min(q_2,p_2)[$.

(16) $]q_1,q_2] \cap]p_1,p_2] =]\max(q_1,p_1),\min(q_2,p_2)]$.

(17) $]p,q[\subseteq [p,q[$ and $]p,q[\subseteq]p,q]$ and $[p,q[\subseteq [p,q]$ and $]p,q] \subseteq [p,q]$.

(18) If $r \in [p,g[$ and $s \in [p,g[$, then $[r,s] \subseteq [p,g[$.

(19) If $r \in]p,g]$ and $s \in]p,g]$, then $[r,s] \subseteq]p,g]$.

(20) If $p \leq q$ and $q \leq r$, then $[p,q] \cup]q,r] = [p,r]$.

(21) If $p \leq q$ and $q \leq r$, then $[p,q[\cup [q,r] = [p,r]$.

(22) If $[q_1,q_2[$ meets $[p_1,p_2[$, then $q_2 \geq p_1$.

(23) If $]q_1,q_2]$ meets $]p_1,p_2]$, then $q_2 \geq p_1$.

(24) If $[q_1,q_2[$ meets $[p_1,p_2[$, then $[q_1,q_2[\cup [p_1,p_2[= [\min(q_1,p_1),\max(q_2,p_2)[$.

(25) If $]q_1,q_2]$ meets $]p_1,p_2]$, then $]q_1,q_2] \cup]p_1,p_2] =]\min(q_1,p_1),\max(q_2,p_2)]$.

(26) If $[p_1,p_2[$ meets $[q_1,q_2[$, then $[p_1,p_2[\setminus [q_1,q_2[= [p_1,q_1[\cup [q_2,p_2[$.

(27) If $]p_1,p_2]$ meets $]q_1,q_2]$, then $]p_1,p_2] \setminus]q_1,q_2] =]p_1,q_1] \cup]q_2,p_2]$.

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