

# Finite Sums of Vectors in Right Module over Associative Ring

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The articles [5], [1], [8], [2], [6], [3], [7], and [4] provide the notation and terminology for this paper.

For simplicity, we use the following convention:  $R$  denotes a ring,  $a$  denotes a scalar of  $R$ ,  $V$  denotes a right module over  $R$ ,  $v, w, u$  denote vectors of  $V$ ,  $F, G$  denote finite sequences of elements of the carrier of  $V$ , and  $k$  denotes a natural number.

One can prove the following propositions:

(9)<sup>1</sup> If  $\text{len } F = \text{len } G$  and for all  $k, v$  such that  $k \in \text{dom } F$  and  $v = G(k)$  holds  $F(k) = v \cdot a$ , then  $\sum F = \sum G \cdot a$ .

(10) If  $\text{len } F = \text{len } G$  and for every  $k$  such that  $k \in \text{dom } F$  holds  $G(k) = F_k \cdot a$ , then  $\sum G = \sum F \cdot a$ .

(21)<sup>2</sup>  $\sum(\varepsilon_{(\text{the carrier of } V)}) \cdot a = 0_V$ .

(23)<sup>3</sup>  $\sum\langle v, u \rangle \cdot a = v \cdot a + u \cdot a$ .

(24)  $\sum\langle v, u, w \rangle \cdot a = v \cdot a + u \cdot a + w \cdot a$ .

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<sup>1</sup> The propositions (1)–(8) have been removed.

<sup>2</sup> The propositions (11)–(20) have been removed.

<sup>3</sup> The proposition (22) has been removed.

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