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On the Order on a Special Polygon

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Summary. The goal of the article is to determine the order of the special points defined in [7] on a special polygon. We restrict ourselves to the clockwise oriented finite sequences (the concept defined in this article) that start in N-min C (C being a compact non empty subset of the plane).

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The articles [16], [20], [2], [18], [5], [6], [3], [19], [4], [17], [1], [14], [15], [8], [9], [10], [11], [13], [12], and [7] provide the notation and terminology for this paper.

1. Preliminaries

The following propositions are true:

(1) For all sets A, B, C, p such that $A \cap B \subseteq \{p\}$ and $p \in C$ and C misses B holds $A \cup C$ misses B.

(2) For all sets A, B, C, p such that $A \cap C = \{p\}$ and $p \in B$ and $B \subseteq C$ holds $A \cap B = \{p\}$.

(4)[1] For all sets A, B such that for all sets x, y such that $x \in A$ and $y \in B$ holds x misses y holds $\bigcup A$ misses $\bigcup B$.

2. On the finite sequences

We follow the rules: i, j, k, m, n denote natural numbers, D denotes a non empty set, and f denotes a finite sequence of elements of D.

We now state several propositions:

(5) If $i \leq j$ and $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $k \in \operatorname{dom} \operatorname{mid}(f, i, j)$, then $(k + i) -' 1 \in \operatorname{dom} f$.

(6) If $i > j$ and $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $k \in \operatorname{dom} \operatorname{mid}(f, i, j)$, then $(i -' k) + 1 \in \operatorname{dom} f$.

(7) If $i \leq j$ and $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $k \in \operatorname{dom} \operatorname{mid}(f, i, j)$, then $(\operatorname{mid}(f, i, j))_k = f_{(k+i) -' 1}$.

(8) If $i > j$ and $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $k \in \operatorname{dom} \operatorname{mid}(f, i, j)$, then $(\operatorname{mid}(f, i, j))_k = f_{(i -' k) + 1}$.

(9) If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$, then $\operatorname{len} \operatorname{mid}(f, i, j) \geq 1$.

[1] The proposition (3) has been removed.

(10) If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{len} \operatorname{mid}(f,i,j) = 1$, then $i = j$.

(11) If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$, then $\operatorname{mid}(f,i,j)$ is non empty.

(12) If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$, then $(\operatorname{mid}(f,i,j))_1 = f_i$.

(13) If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$, then $(\operatorname{mid}(f,i,j))_{\operatorname{len} \operatorname{mid}(f,i,j)} = f_j$.

3. Compact subsets of the plane

In the sequel X denotes a compact subset of $\mathcal{E}^2_{\mathrm{T}}$.

One can prove the following four propositions:

(14) For every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in X$ and $p_{\mathbf{2}} = \text{N-bound}(X)$ holds $p \in \mathrm{N}_{\mathrm{most}}(X)$.

(15) For every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in X$ and $p_{\mathbf{2}} = \text{S-bound}(X)$ holds $p \in \mathrm{S}_{\mathrm{most}}(X)$.

(16) For every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in X$ and $p_{\mathbf{1}} = \text{W-bound}(X)$ holds $p \in \mathrm{W}_{\mathrm{most}}(X)$.

(17) For every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in X$ and $p_{\mathbf{1}} = \text{E-bound}(X)$ holds $p \in \mathrm{E}_{\mathrm{most}}(X)$.

4. Finite sequences on the plane

One can prove the following propositions:

(18) For every finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $1 \leq i$ and $i \leq j$ and $j \leq \operatorname{len} f$ holds $\widetilde{\mathcal{L}}(\operatorname{mid}(f,i,j)) = \bigcup\{\mathcal{L}(f,k) : i \leq k \wedge k < j\}$.

(19) For every finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\operatorname{dom} \mathbf{X}\text{-coordinate}(f) = \operatorname{dom} f$.

(20) For every finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\operatorname{dom} \mathbf{Y}\text{-coordinate}(f) = \operatorname{dom} f$.

(21) For all points a, b, c of $\mathcal{E}^2_{\mathrm{T}}$ such that $b \in \mathcal{L}(a,c)$ and $a_{\mathbf{1}} \leq b_{\mathbf{1}}$ and $c_{\mathbf{1}} \leq b_{\mathbf{1}}$ holds $a = b$ or $b = c$ or $a_{\mathbf{1}} = b_{\mathbf{1}}$ and $c_{\mathbf{1}} = b_{\mathbf{1}}$.

(22) For all points a, b, c of $\mathcal{E}^2_{\mathrm{T}}$ such that $b \in \mathcal{L}(a,c)$ and $a_{\mathbf{2}} \leq b_{\mathbf{2}}$ and $c_{\mathbf{2}} \leq b_{\mathbf{2}}$ holds $a = b$ or $b = c$ or $a_{\mathbf{2}} = b_{\mathbf{2}}$ and $c_{\mathbf{2}} = b_{\mathbf{2}}$.

(23) For all points a, b, c of $\mathcal{E}^2_{\mathrm{T}}$ such that $b \in \mathcal{L}(a,c)$ and $a_{\mathbf{1}} \geq b_{\mathbf{1}}$ and $c_{\mathbf{1}} \geq b_{\mathbf{1}}$ holds $a = b$ or $b = c$ or $a_{\mathbf{1}} = b_{\mathbf{1}}$ and $c_{\mathbf{1}} = b_{\mathbf{1}}$.

(24) For all points a, b, c of $\mathcal{E}^2_{\mathrm{T}}$ such that $b \in \mathcal{L}(a,c)$ and $a_{\mathbf{2}} \geq b_{\mathbf{2}}$ and $c_{\mathbf{2}} \geq b_{\mathbf{2}}$ holds $a = b$ or $b = c$ or $a_{\mathbf{2}} = b_{\mathbf{2}}$ and $c_{\mathbf{2}} = b_{\mathbf{2}}$.

5. The area of a sequence

Let f, g be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. We say that g is in the area of f if and only if:

(Def. 1) For every n such that $n \in \operatorname{dom} g$ holds $\text{W-bound}(\widetilde{\mathcal{L}}(f)) \leq (g_n)_{\mathbf{1}}$ and $(g_n)_{\mathbf{1}} \leq \text{E-bound}(\widetilde{\mathcal{L}}(f))$ and $\text{S-bound}(\widetilde{\mathcal{L}}(f)) \leq (g_n)_{\mathbf{2}}$ and $(g_n)_{\mathbf{2}} \leq \text{N-bound}(\widetilde{\mathcal{L}}(f))$.

The following propositions are true:

(25) Every non trivial finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ is in the area of f.

(26) Let f, g be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose g is in the area of f. Let given i, j. If $i \in \operatorname{dom} g$ and $j \in \operatorname{dom} g$, then $\operatorname{mid}(g,i,j)$ is in the area of f.

(27) Let f be a non trivial finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and given i, j. If $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$, then $\operatorname{mid}(f,i,j)$ is in the area of f.

(28) Let f, g, h be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose g is in the area of f and h is in the area of f. Then $g \frown h$ is in the area of f.

(29) For every non trivial finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\langle \text{NE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is in the area of f.

(30) For every non trivial finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\langle \text{NW-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is in the area of f.

(31) For every non trivial finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\langle \text{SE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is in the area of f.

(32) For every non trivial finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\langle \text{SW-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is in the area of f.

6. HORIZONTAL AND VERTICAL CONNECTIONS

Let f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. We say that g is a h.c. for f if and only if:

(Def. 2) g is in the area of f and $(g_1)_{\mathbf{1}} = \text{W-bound}(\widetilde{\mathcal{L}}(f))$ and $(g_{\text{len}\, g})_{\mathbf{1}} = \text{E-bound}(\widetilde{\mathcal{L}}(f))$.

We say that g is a v.c. for f if and only if:

(Def. 3) g is in the area of f and $(g_1)_{\mathbf{2}} = \text{S-bound}(\widetilde{\mathcal{L}}(f))$ and $(g_{\text{len}\, g})_{\mathbf{2}} = \text{N-bound}(\widetilde{\mathcal{L}}(f))$.

Next we state the proposition

(33) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and g, h be one-to-one special finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose $2 \leq \text{len}\, g$ and $2 \leq \text{len}\, h$ and g is a h.c. for f and h is a v.c. for f. Then $\widetilde{\mathcal{L}}(g)$ meets $\widetilde{\mathcal{L}}(h)$.

7. ORIENTATION

Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$. We say that f is clockwise oriented if and only if:

(Def. 4) $(f \circlearrowleft \text{N}_{\text{min}}(\widetilde{\mathcal{L}}(f)))_2 \in \text{N}_{\text{most}}(\widetilde{\mathcal{L}}(f))$.

Next we state the proposition

(34) Let f be a non constant standard special circular sequence. If $f_1 = \text{N}_{\text{min}}(\widetilde{\mathcal{L}}(f))$, then f is clockwise oriented iff $f_2 \in \text{N}_{\text{most}}(\widetilde{\mathcal{L}}(f))$.

Let us observe that $\square_{\mathcal{E}^2}$ is compact.
Next we state several propositions:

(35) $\text{N-bound}(\square_{\mathcal{E}^2}) = 1$.

(36) $\text{W-bound}(\square_{\mathcal{E}^2}) = 0$.

(37) $\text{E-bound}(\square_{\mathcal{E}^2}) = 1$.

(38) $\text{S-bound}(\square_{\mathcal{E}^2}) = 0$.

(39) $\text{N}_{\text{most}}(\square_{\mathcal{E}^2}) = \mathcal{L}([0,1],[1,1])$.

(40) $\text{N}_{\text{min}}(\square_{\mathcal{E}^2}) = [0,1]$.

Let X be a non vertical non horizontal non empty compact subset of $\mathcal{E}^2_{\rm T}$. Note that SpStSeq X is clockwise oriented.

One can verify that there exists a non constant standard special circular sequence which is clockwise oriented.

We now state two propositions:

(41) Let f be a non constant standard special circular sequence and given i, j. Suppose $i > j$ but $1 < j$ and $i \leq \text{len}\, f$ or $1 \leq j$ and $i < \text{len}\, f$. Then $\text{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$.

(42) Let f be a non constant standard special circular sequence and given i, j. Suppose $i < j$ but $1 < i$ and $j \leq \text{len}\, f$ or $1 \leq i$ and $j < \text{len}\, f$. Then $\text{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$.

In the sequel f denotes a non trivial finite sequence of elements of $\mathcal{E}^2_{\rm T}$.
The following propositions are true:

(43) $\text{N}_{\min}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(44) $\text{N}_{\max}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(45) $\text{S}_{\min}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(46) $\text{S}_{\max}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(47) $\text{W}_{\min}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(48) $\text{W}_{\max}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(49) $\text{E}_{\min}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

(50) $\text{E}_{\max}(\widetilde{\mathcal{L}}(f)) \in \text{rng}\, f$.

In the sequel f denotes a non constant standard special circular sequence.
We now state a number of propositions:

(51) If $1 \leq i$ and $i \leq j$ and $j < m$ and $m \leq n$ and $n \leq \text{len}\, f$ and $1 < i$ or $n < \text{len}\, f$, then $\widetilde{\mathcal{L}}(\text{mid}(f,i,j))$ misses $\widetilde{\mathcal{L}}(\text{mid}(f,m,n))$.

(52) If $1 \leq i$ and $i \leq j$ and $j < m$ and $m \leq n$ and $n \leq \text{len}\, f$ and $1 < i$ or $n < \text{len}\, f$, then $\widetilde{\mathcal{L}}(\text{mid}(f,i,j))$ misses $\widetilde{\mathcal{L}}(\text{mid}(f,n,m))$.

(53) If $1 \leq i$ and $i \leq j$ and $j < m$ and $m \leq n$ and $n \leq \text{len}\, f$ and $1 < i$ or $n < \text{len}\, f$, then $\widetilde{\mathcal{L}}(\text{mid}(f,j,i))$ misses $\widetilde{\mathcal{L}}(\text{mid}(f,n,m))$.

(54) If $1 \leq i$ and $i \leq j$ and $j < m$ and $m \leq n$ and $n \leq \text{len}\, f$ and $1 < i$ or $n < \text{len}\, f$, then $\widetilde{\mathcal{L}}(\text{mid}(f,j,i))$ misses $\widetilde{\mathcal{L}}(\text{mid}(f,m,n))$.

(55) $(\text{N}_{\min}(\widetilde{\mathcal{L}}(f)))_{\mathbf{1}} < (\text{N}_{\max}(\widetilde{\mathcal{L}}(f)))_{\mathbf{1}}$.

(56) $\text{N}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{N}_{\max}(\widetilde{\mathcal{L}}(f))$.

(57) $(\text{E}_{\min}(\widetilde{\mathcal{L}}(f)))_{\mathbf{2}} < (\text{E}_{\max}(\widetilde{\mathcal{L}}(f)))_{\mathbf{2}}$.

(58) $\text{E}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{E}_{\max}(\widetilde{\mathcal{L}}(f))$.

(59) $(\text{S}_{\min}(\widetilde{\mathcal{L}}(f)))_{\mathbf{1}} < (\text{S}_{\max}(\widetilde{\mathcal{L}}(f)))_{\mathbf{1}}$.

(60) $\text{S}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{S}_{\max}(\widetilde{\mathcal{L}}(f))$.

(61) $(\text{W}_{\min}(\widetilde{\mathcal{L}}(f)))_{\mathbf{2}} < (\text{W}_{\max}(\widetilde{\mathcal{L}}(f)))_{\mathbf{2}}$.

(62) $\text{W}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{W}_{\max}(\widetilde{\mathcal{L}}(f))$.

(63) $\mathcal{L}(\text{NW-corner}(\widetilde{\mathcal{L}}(f)), \text{N}_{\min}(\widetilde{\mathcal{L}}(f)))$ misses $\mathcal{L}(\text{N}_{\max}(\widetilde{\mathcal{L}}(f)), \text{NE-corner}(\widetilde{\mathcal{L}}(f)))$.

(64) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. Suppose f is a special sequence and $p \neq f_1$ and $p_{\mathbf{1}} = (f_1)_{\mathbf{1}}$ or $p_{\mathbf{2}} = (f_1)_{\mathbf{2}}$ and $\mathcal{L}(p, f_1) \cap \widetilde{\mathcal{L}}(f) = \{f_1\}$. Then $\langle p \rangle \frown f$ is a S-sequence in $\mathbb{R}^2$.

(65) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. Suppose f is a special sequence and $p \neq f_{\text{len}\, f}$ and $p_{\mathbf{1}} = (f_{\text{len}\, f})_{\mathbf{1}}$ or $p_{\mathbf{2}} = (f_{\text{len}\, f})_{\mathbf{2}}$ and $\mathcal{L}(p, f_{\text{len}\, f}) \cap \widetilde{\mathcal{L}}(f) = \{f_{\text{len}\, f}\}$. Then $f \frown \langle p \rangle$ is a S-sequence in $\mathbb{R}^2$.

8. APPENDING CORNERS

The following propositions are true:

(66) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_j = \mathrm{N}_{\max}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(f)) \neq \text{NE-corner}(\widetilde{\mathcal{L}}(f))$. Then $(\operatorname{mid}(f,i,j)) \frown \langle \text{NE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is a S-sequence in $\mathbb{R}^2$.

(67) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_j = \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{E}_{\max}(\widetilde{\mathcal{L}}(f)) \neq \text{NE-corner}(\widetilde{\mathcal{L}}(f))$. Then $(\operatorname{mid}(f,i,j)) \frown \langle \text{NE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is a S-sequence in $\mathbb{R}^2$.

(68) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_j = \mathrm{S}_{\max}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{S}_{\max}(\widetilde{\mathcal{L}}(f)) \neq \text{SE-corner}(\widetilde{\mathcal{L}}(f))$. Then $(\operatorname{mid}(f,i,j)) \frown \langle \text{SE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is a S-sequence in $\mathbb{R}^2$.

(69) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_j = \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{E}_{\max}(\widetilde{\mathcal{L}}(f)) \neq \text{NE-corner}(\widetilde{\mathcal{L}}(f))$. Then $(\operatorname{mid}(f,i,j)) \frown \langle \text{NE-corner}(\widetilde{\mathcal{L}}(f)) \rangle$ is a S-sequence in $\mathbb{R}^2$.

(70) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_i = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{NW-corner}(\widetilde{\mathcal{L}}(f))$. Then $\langle \text{NW-corner}(\widetilde{\mathcal{L}}(f)) \rangle \frown \operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$.

(71) Let given i, j. Suppose $i \in \operatorname{dom} f$ and $j \in \operatorname{dom} f$ and $\operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$ and $f_i = \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(f))$ and $\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(f)) \neq \text{SW-corner}(\widetilde{\mathcal{L}}(f))$. Then $\langle \text{SW-corner}(\widetilde{\mathcal{L}}(f)) \rangle \frown \operatorname{mid}(f,i,j)$ is a S-sequence in $\mathbb{R}^2$.

Let f be a non constant standard special circular sequence. Note that $\widetilde{\mathcal{L}}(f)$ satisfies conditions of simple closed curve.

9. THE ORDER

Next we state two propositions:

(72) If $f_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f))$, then $(\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f))) \leftarrow f < (\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(f))) \leftarrow f$.

(73) If $f_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f))$, then $(\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(f))) \leftarrow f > 1$.

In the sequel z is a clockwise oriented non constant standard special circular sequence.

The following propositions are true:

(74) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$ and $\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(z)) \neq \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{E}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(75) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{E}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{E}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(76) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$ and $\mathrm{E}_{\min}(\widetilde{\mathcal{L}}(z)) \neq \mathrm{S}_{\max}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{E}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{S}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(77) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{S}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{S}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(78) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$ and $\mathrm{S}_{\min}(\widetilde{\mathcal{L}}(z)) \neq \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{S}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(79) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$ and $\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z)) \neq \mathrm{W}_{\max}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z < (\mathrm{W}_{\max}(\widetilde{\mathcal{L}}(z))) \leftarrow z$.

(80) If $z_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(z))$, then $(\mathrm{W}_{\min}(\widetilde{\mathcal{L}}(z))) \leftarrow z < \operatorname{len} z$.

(81) If $f_1 = \mathrm{N}_{\min}(\widetilde{\mathcal{L}}(f))$, then $(\mathrm{W}_{\max}(\widetilde{\mathcal{L}}(f))) \leftarrow f < \operatorname{len} f$.

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