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Complex Numbers — Basic Definitions

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The articles [8], [10], [5], [3], [4], [6], [1], [2], [9], and [7] provide the notation and terminology for this paper.

The functor i is defined by:

(Def. 1) $i = [0 \longmapsto 0, 1 \longmapsto 1]$.

Let c be a number. We say that c is complex if and only if:

(Def. 2) $c \in \mathbb{C}$.

Let us note that i is complex.

Let us mention that there exists a number which is complex.

Let x be a complex number. Let us observe that x is empty if and only if:

(Def. 3) $x = 0$.

We introduce x is zero as a synonym of x is empty.

Let x, y be complex numbers. The functor $x + y$ is defined as follows:

(Def. 4) There exist elements x_1, x_2, y_1, y_2 of $\mathbb{R}$ such that $x = x_1 + x_2 i$ and $y = y_1 + y_2 i$ and $x + y = +(x_1, y_1) + +(x_2, y_2)i$.

Let us note that the functor $x + y$ is commutative. The functor $x \cdot y$ is defined by:

(Def. 5) There exist elements x_1, x_2, y_1, y_2 of $\mathbb{R}$ such that $x = x_1 + x_2 i$ and $y = y_1 + y_2 i$ and $x \cdot y = +(\cdot(x_1, y_1), {}^{\mathrm{op}}\cdot(x_2, y_2)) + +(\cdot(x_1, y_2), \cdot(x_2, y_1))i$.

Let us observe that the functor $x \cdot y$ is commutative.

Let z, z' be complex numbers. Observe that $z + z'$ is complex and $z \cdot z'$ is complex.

Let z be a complex number. The functor $-z$ yields a complex number and is defined as follows:

(Def. 6) $z + -z = 0$.

Let us notice that the functor $-z$ is involutive. The functor z^{-1} yielding a complex number is defined by:

(Def. 7)(i) $z \cdot z^{-1} = 1$ if $z \neq 0$,

(ii) $z^{-1} = 0$, otherwise.

Let us observe that the functor z^{-1} is involutive.

Let x, y be complex numbers. The functor $x - y$ is defined by:

(Def. 8) $x - y = x + -y$.

The functor $\frac{x}{y}$ is defined by:

(Def. 9) $\frac{x}{y} = x \cdot y^{-1}$.

Let x, y be complex numbers. Note that $x - y$ is complex and $\frac{x}{y}$ is complex.

One can verify that there exists a complex number which is non zero.

Let x be a non zero complex number. One can verify that $-x$ is non zero and x^{-1} is non zero. Let y be a non zero complex number. One can verify that $x \cdot y$ is non zero.

Let x, y be non zero complex numbers. Observe that $\frac{x}{y}$ is non zero.

Let us observe that every element of $\mathbb{R}$ is complex.

Let us observe that every number which is natural is also complex.

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