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The Jordan's Property for Certain Subsets of the Plane

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Summary. Let S be a subset of the topological Euclidean plane $\mathcal{E}^2_{\rm T}$. We say that S has Jordan's property if there exist two non-empty, disjoint and connected subsets G_1 and G_2 of $\mathcal{E}^2_{\rm T}$ such that $S^{\rm c} = G_1 \cup G_2$ and $\overline{G_1} \setminus G_1 = \overline{G_2} \setminus G_2$ (see [19], [10]). The aim is to prove that the boundaries of some special polygons in $\mathcal{E}^2_{\rm T}$ have this property (see Section 3). Moreover, it is proved that both the interior and the exterior of the boundary of any rectangle in $\mathcal{E}^2_{\rm T}$ is open and connected.

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The articles [22], [24], [11], [17], [1], [4], [5], [20], [3], [16], [7], [15], [23], [18], [12], [2], [21], [14], [13], [8], [6], and [9] provide the notation and terminology for this paper.

1. Selected theorems on connected spaces

In the sequel G_1, G_2 are topological spaces and A is a subset of G_1. The following propositions are true:

(1) If $A \neq \emptyset$, then the carrier of $G_1 \restriction A = A$.

(2) For every topological space G_1 if for every points x, y of G_1 there exists G_2 such that G_2 is connected and there exists a map f from G_2 into G_1 such that f is continuous and $x \in \operatorname{rng} f$ and $y \in \operatorname{rng} f$, then G_1 is connected.

The following propositions are true:

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(3) For every topological space G_1 if for all points x, y of G_1 such that $x \neq y$ there exists a map h from $\mathbb{I}$ into G_1 such that h is continuous and $x = h(0)$ and $y = h(1)$, then G_1 is connected.

(4) Let A be a subset of G_1. Then if $A \neq \emptyset_{G_1}$ and for all points x_1, y_1 of G_1 such that $x_1 \in A$ and $y_1 \in A$ and $x_1 \neq y_1$ there exists a map h from $\mathbb{I}$ into $G_1 \restriction A$ such that h is continuous and $x_1 = h(0)$ and $y_1 = h(1)$, then A is connected.

(5) For every G_1 and for every subset A_0 of G_1 and for every subset A_1 of G_1 such that A_0 is connected and A_1 is connected and $A_0 \cap A_1 \neq \emptyset$ holds $A_0 \cup A_1$ is connected.

(6) For every G_1 and for all subsets A_0, A_1, A_2 of G_1 such that A_0 is connected and A_1 is connected and A_2 is connected and $A_0 \cap A_1 \neq \emptyset$ and $A_1 \cap A_2 \neq \emptyset$ holds $A_0 \cup A_1 \cup A_2$ is connected.

(7) For every G_1 and for all subsets A_0, A_1, A_2, A_3 of G_1 such that A_0 is connected and A_1 is connected and A_2 is connected and A_3 is connected and $A_0 \cap A_1 \neq \emptyset$ and $A_1 \cap A_2 \neq \emptyset$ and $A_2 \cap A_3 \neq \emptyset$ holds $A_0 \cup A_1 \cup A_2 \cup A_3$ is connected.

2. Certain connected and open subsets in the Euclidean plane

We follow a convention: P, Q, P_1, P_2 denote subsets of $\mathcal{E}^2_{\mathrm{T}}$ and w_1, w_2 denote points of $\mathcal{E}^2_{\mathrm{T}}$. One can prove the following proposition

(8) For every P such that $P \neq \emptyset_{\mathcal{E}^2_{\mathrm{T}}}$ and for all w_1, w_2 such that $w_1 \in P$ and $w_2 \in P$ and $w_1 \neq w_2$ holds $\mathcal{L}(w_1, w_2) \subseteq P$ holds P is connected.

We adopt the following rules: p_1, p_2 will be points of $\mathcal{E}^2_{\mathrm{T}}$ and s_1, t_1, s_2, t_2, s, t, s_3, t_3, s_4, t_4, s_5, t_5, s_6, t_6, l, s_7, t_7 will be real numbers. Next we state two propositions:

(9) If $s_1 < s_3$ and $s_1 < s_4$ and $0 \leq l$ and $l \leq 1$, then $s_1 < (1-l) \cdot s_3 + l \cdot s_4$.

(10) If $s_3 < s_1$ and $s_4 < s_1$ and $0 \leq l$ and $l \leq 1$, then $(1-l) \cdot s_3 + l \cdot s_4 < s_1$.

In the sequel s_8, t_8 denote real numbers. The following propositions are true:

(11) $\{[s,t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\} = \{[s_3,t_3] : s_1 < s_3\} \cap \{[s_4, t_4] : s_4 < s_2\} \cap \{[s_5,t_5] : t_1 < t_5\} \cap \{[s_6,t_6] : t_6 < t_2\}$.

(12) $\{[s,t] : \neg(s_1 \leq s \wedge s \leq s_2 \wedge t_1 \leq t \wedge t \leq t_2)\} = \{[s_3,t_3] : s_3 < s_1\} \cup \{[s_4, t_4] : t_4 < t_1\} \cup \{[s_5,t_5] : s_2 < s_5\} \cup \{[s_6,t_6] : t_2 < t_6\}$.

(13) For all s_1, t_1, s_2, t_2, P such that $s_1 < s_2$ and $t_1 < t_2$ and $P = \{[s, t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\}$ holds P is connected.

(14) For all s_1, P such that $P = \{[s,t] : s_1 < s\}$ holds P is connected.

(15) For all s_2, P such that $P = \{[s,t] : s < s_2\}$ holds P is connected.

(16) For all t_1, P such that $P = \{[s,t] : t_1 < t\}$ holds P is connected.

(17) For all t_2, P such that $P = \{[s,t] : t < t_2\}$ holds P is connected.

(18) For all s_1, t_1, s_2, t_2, P such that $P = \{[s, t] : \neg(s_1 \leq s \wedge s \leq s_2 \wedge t_1 \leq t \wedge t \leq t_2)\}$ holds P is connected.

(19) For all s_1, P such that $P = \{[s, t] : s_1 < s\}$ holds P is open.

(20) For all s_1, P such that $P = \{[s, t] : s_1 > s\}$ holds P is open.

(21) For all s_1, P such that $P = \{[s, t] : s_1 < t\}$ holds P is open.

(22) For all s_1, P such that $P = \{[s, t] : s_1 > t\}$ holds P is open.

(23) For all s_1, t_1, s_2, t_2, P such that $P = \{[s, t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\}$ holds P is open.

(24) For all s_1, t_1, s_2, t_2, P such that $P = \{[s, t] : \neg(s_1 \leq s \wedge s \leq s_2 \wedge t_1 \leq t \wedge t \leq t_2)\}$ holds P is open.

(25) Given s_1, t_1, s_2, t_2, P, Q. Suppose $P = \{[s_7, t_7] : s_1 < s_7 \wedge s_7 < s_2 \wedge t_1 < t_7 \wedge t_7 < t_2\}$ and $Q = \{[s_8, t_8] : \neg(s_1 \leq s_8 \wedge s_8 \leq s_2 \wedge t_1 \leq t_8 \wedge t_8 \leq t_2)\}$. Then $P \cap Q = \emptyset_{\mathcal{E}^2_{\rm T}}$.

(26) For all real numbers s_1, s_2, t_1, t_2 holds $\{p : s_1 < p_{\mathbf{1}} \wedge p_{\mathbf{1}} < s_2 \wedge t_1 < p_{\mathbf{2}} \wedge p_{\mathbf{2}} < t_2\} = \{[s_7, t_7] : s_1 < s_7 \wedge s_7 < s_2 \wedge t_1 < t_7 \wedge t_7 < t_2\}$, where p ranges over points of $\mathcal{E}^2_{\rm T}$.

(27) For all s_1, s_2, t_1, t_2 holds $\{q_1 : \neg(s_1 \leq q_{1\mathbf{1}} \wedge q_{1\mathbf{1}} \leq s_2 \wedge t_1 \leq q_{1\mathbf{2}} \wedge q_{1\mathbf{2}} \leq t_2)\} = \{[s_8, t_8] : \neg(s_1 \leq s_8 \wedge s_8 \leq s_2 \wedge t_1 \leq t_8 \wedge t_8 \leq t_2)\}$, where q_1 ranges over points of $\mathcal{E}^2_{\rm T}$.

(28) For all s_1, s_2, t_1, t_2 holds $\{p_0 : s_1 < p_{0\mathbf{1}} \wedge p_{0\mathbf{1}} < s_2 \wedge t_1 < p_{0\mathbf{2}} \wedge p_{0\mathbf{2}} < t_2\}$, where p_0 ranges over points of $\mathcal{E}^2_{\rm T}$, is a subset of $\mathcal{E}^2_{\rm T}$.

(29) For all s_1, s_2, t_1, t_2 holds $\{p_3 : \neg(s_1 \leq p_{3\mathbf{1}} \wedge p_{3\mathbf{1}} \leq s_2 \wedge t_1 \leq p_{3\mathbf{2}} \wedge p_{3\mathbf{2}} \leq t_2)\}$, where p_3 ranges over points of $\mathcal{E}^2_{\rm T}$, is a subset of $\mathcal{E}^2_{\rm T}$.

(30) For all s_1, t_1, s_2, t_2, P such that $s_1 < s_2$ and $t_1 < t_2$ and $P = \{p_0 : s_1 < p_{0\mathbf{1}} \wedge p_{0\mathbf{1}} < s_2 \wedge t_1 < p_{0\mathbf{2}} \wedge p_{0\mathbf{2}} < t_2\}$, where p_0 ranges over points of $\mathcal{E}^2_{\rm T}$ holds P is connected.

(31) For all s_1, t_1, s_2, t_2, P such that $P = \{p_3 : \neg(s_1 \leq p_{3\mathbf{1}} \wedge p_{3\mathbf{1}} \leq s_2 \wedge t_1 \leq p_{3\mathbf{2}} \wedge p_{3\mathbf{2}} \leq t_2)\}$, where p_3 ranges over points of $\mathcal{E}^2_{\rm T}$ holds P is connected.

(32) For all s_1, t_1, s_2, t_2, P such that $P = \{p_0 : s_1 < p_{0\mathbf{1}} \wedge p_{0\mathbf{1}} < s_2 \wedge t_1 < p_{0\mathbf{2}} \wedge p_{0\mathbf{2}} < t_2\}$, where p_0 ranges over points of $\mathcal{E}^2_{\rm T}$ holds P is open.

(33) For all s_1, t_1, s_2, t_2, P such that $P = \{p_3 : \neg(s_1 \leq p_{3\mathbf{1}} \wedge p_{3\mathbf{1}} \leq s_2 \wedge t_1 \leq p_{3\mathbf{2}} \wedge p_{3\mathbf{2}} \leq t_2)\}$, where p_3 ranges over points of $\mathcal{E}^2_{\rm T}$ holds P is open.

(34) Given s_1, t_1, s_2, t_2, P, Q. Suppose $P = \{p : s_1 < p_{\mathbf{1}} \wedge p_{\mathbf{1}} < s_2 \wedge t_1 < p_{\mathbf{2}} \wedge p_{\mathbf{2}} < t_2\}$, where p ranges over points of $\mathcal{E}^2_{\rm T}$ and $Q = \{q_1 : \neg(s_1 \leq q_{1\mathbf{1}} \wedge q_{1\mathbf{1}} \leq s_2 \wedge t_1 \leq q_{1\mathbf{2}} \wedge q_{1\mathbf{2}} \leq t_2)\}$, where q_1 ranges over points of $\mathcal{E}^2_{\rm T}$. Then $P \cap Q = \emptyset_{\mathcal{E}^2_{\rm T}}$.

(35) Given s_1, t_1, s_2, t_2, P, P_1, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \ \vee\ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \ \vee\ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \ \vee\ p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\rm T}$,

(iv) $P_1 = \{p_1 : s_1 < p_{1\mathbf{1}} \land p_{1\mathbf{1}} < s_2 \land t_1 < p_{1\mathbf{2}} \land p_{1\mathbf{2}} < t_2\}$, where p_1 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(v) $P_2 = \{p_2 : \neg(s_1 \leq p_{2\mathbf{1}} \land p_{2\mathbf{1}} \leq s_2 \land t_1 \leq p_{2\mathbf{2}} \land p_{2\mathbf{2}} \leq t_2)\}$, where p_2 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then

(vi) $P^{\mathrm{c}} = P_1 \cup P_2$,

(vii) $P^{\mathrm{c}} \neq \emptyset$,

(viii) $P_1 \cap P_2 = \emptyset$,

(ix) for all subsets P_3, P_4 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$ such that $P_3 = P_1$ and $P_4 = P_2$ holds P_3 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$ and P_4 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$.

(36) Given s_1, t_1, s_2, t_2, P, P_1, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_2 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_1 \ \lor \ p_{\mathbf{1}} = s_2 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(iv) $P_1 = \{p_1 : s_1 < p_{1\mathbf{1}} \land p_{1\mathbf{1}} < s_2 \land t_1 < p_{1\mathbf{2}} \land p_{1\mathbf{2}} < t_2\}$, where p_1 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(v) $P_2 = \{p_2 : \neg(s_1 \leq p_{2\mathbf{1}} \land p_{2\mathbf{1}} \leq s_2 \land t_1 \leq p_{2\mathbf{2}} \land p_{2\mathbf{2}} \leq t_2)\}$, where p_2 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then $P = \overline{P_1} \setminus P_1$ and $P = \overline{P_2} \setminus P_2$.

(37) Given s_1, s_2, t_1, t_2, P, P_1. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_2 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_1 \ \lor \ p_{\mathbf{1}} = s_2 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(iv) $P_1 = \{p_1 : s_1 < p_{1\mathbf{1}} \land p_{1\mathbf{1}} < s_2 \land t_1 < p_{1\mathbf{2}} \land p_{1\mathbf{2}} < t_2\}$, where p_1 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then $P_1 \subseteq \Omega_{(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}}$.

(38) Given s_1, s_2, t_1, t_2, P, P_1. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_2 \ \lor \ p_{\mathbf{1}} \leq s_2 \land p_{\mathbf{1}} \geq s_1 \land p_{\mathbf{2}} = t_1 \ \lor \ p_{\mathbf{1}} = s_2 \land p_{\mathbf{2}} \leq t_2 \land p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(iv) $P_1 = \{p_1 : s_1 < p_{1\mathbf{1}} \land p_{1\mathbf{1}} < s_2 \land t_1 < p_{1\mathbf{2}} \land p_{1\mathbf{2}} < t_2\}$, where p_1 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then P_1 is a subset of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$.

(39) Given s_1, s_2, t_1, t_2, P, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \ \vee \ p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(iv) $P_2 = \{p_2 : \neg(s_1 \leq p_{2\mathbf{1}} \wedge p_{2\mathbf{1}} \leq s_2 \wedge t_1 \leq p_{2\mathbf{2}} \wedge p_{2\mathbf{2}} \leq t_2)\}$, where p_2 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then $P_2 \subseteq \Omega_{(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}}$.

(40) Given s_1, s_2, t_1, t_2, P, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \ \vee \ p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$,

(iv) $P_2 = \{p_2 : \neg(s_1 \leq p_{2\mathbf{1}} \wedge p_{2\mathbf{1}} \leq s_2 \wedge t_1 \leq p_{2\mathbf{2}} \wedge p_{2\mathbf{2}} \leq t_2)\}$, where p_2 ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then P_2 is a subset of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$.

3. Jordan's property

In the sequel S, A_1, A_2 will be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Let us consider S. We say that S has Jordan's property if and only if the conditions (Def.1) is satisfied.

(Def.1) (i) $S^{\mathrm{c}} \neq \emptyset$,

(ii) there exist A_1, A_2 such that $S^{\mathrm{c}} = A_1 \cup A_2$ and $A_1 \cap A_2 = \emptyset$ and $\overline{A_1} \setminus A_1 = \overline{A_2} \setminus A_2$ and for all subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ such that $C_1 = A_1$ and $C_2 = A_2$ holds C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$.

The following propositions are true:

(41) Suppose S has Jordan's property. Then

(i) $S^{\mathrm{c}} \neq \emptyset$,

(ii) there exist subsets A_1, A_2 of $\mathcal{E}^2_{\mathrm{T}}$ and there exist subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ such that $S^{\mathrm{c}} = A_1 \cup A_2$ and $A_1 \cap A_2 = \emptyset$ and $\overline{A_1} \setminus A_1 = \overline{A_2} \setminus A_2$ and $C_1 = A_1$ and $C_2 = A_2$ and C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ and for every subset C_3 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ such that C_3 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction S^{\mathrm{c}}$ holds $C_3 = C_1$ or $C_3 = C_2$.

(42) Given s_1, s_2, t_1, t_2, P. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p : p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \ \vee \ p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \ \vee \ p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, where p ranges over points of $\mathcal{E}^2_{\mathrm{T}}$.

Then P has Jordan's property.

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