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On T_1 Reflex of Topological Space

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Summary. This article contains a definition of T_1 reflex of a topological space as a quotient space which is T_1 and fulfils the condition that every continuous map f from a topological space T into S being T_1 space can be considered as a superposition of two continuous maps: the first from T onto its T_1 reflex and the last from T_1 reflex of T into S.

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The articles [11], [9], [7], [2], [3], [6], [12], [5], [10], [8], [4], and [1] provide the notation and terminology for this paper.

In this paper X denotes a non empty set and w denotes a set.

One can prove the following propositions:

(1) For every set y and for all functions f, g holds $(f \cdot g)^{-1}(y) = g^{-1}(f^{-1}(y))$.

(2) Let T be a non empty topological space, A be a non empty partition of the carrier of T, and y be a subset of the carrier of the decomposition space of A. Then (the projection onto $A)^{-1}(y) = \bigcup y$.

(3) For every non empty set X and for every partition S of X and for every subset A of S holds $\bigcup S \setminus \bigcup A = \bigcup (S \setminus A)$.

(4) For every non empty set X and for every subset A of X and for every partition S of X such that $A \in S$ holds $\bigcup (S \setminus \{A\}) = X \setminus A$.

(5) Let T be a non empty topological space, S be a non empty partition of the carrier of T, A be a subset of the decomposition space of S, and B be a subset of T. If $B = \bigcup A$, then A is closed iff B is closed.

Let X be a non empty set, let x be an element of X, and let S_1 be a partition of X. The functor $\mathrm{EqClass}(x, S_1)$ yielding a subset of X is defined by:

(Def. 1) $x \in \mathrm{EqClass}(x, S_1)$ and $\mathrm{EqClass}(x, S_1) \in S_1$.

Next we state two propositions:

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(6) For all partitions S_1, S_2 of X such that for every element x of X holds $\mathrm{EqClass}(x, S_1) = \mathrm{EqClass}(x, S_2)$ holds $S_1 = S_2$.

(7) For every non empty set X holds $\{X\}$ is a partition of X.

Let X be a set. Partition family of X is defined by:

(Def. 2) For every set S such that $S \in$ it holds S is a partition of X.

Let X be a non empty set. One can check that there exists a partition of X which is non empty.

One can prove the following proposition

(8) For every set X and for every partition p of X holds $\{p\}$ is a partition family of X.

Let X be a set. One can check that there exists a partition family of X which is non empty.

Next we state two propositions:

(9) For every partition S_1 of X and for all elements x, y of X such that $\mathrm{EqClass}(x, S_1)$ meets $\mathrm{EqClass}(y, S_1)$ holds $\mathrm{EqClass}(x, S_1) = \mathrm{EqClass}(y, S_1)$.

(10) Let A be a set, X be a non empty set, and S be a partition of X. If $A \in S$, then there exists an element x of X such that $A = \mathrm{EqClass}(x, S)$.

Let X be a non empty set and let F be a non empty partition family of X. The functor $\mathrm{Intersection}\, F$ yields a non empty partition of X and is defined as follows:

(Def. 3) For every element x of X holds $\mathrm{EqClass}(x, \mathrm{Intersection}\, F) = \bigcap\{\mathrm{EqClass}(x, S); S$ ranges over partitions of X: $S \in F\}$.

In the sequel T denotes a non empty topological space.

One can prove the following proposition

(11) $\{A; A$ ranges over partitions of the carrier of T: A is closed$\}$ is a partition family of the carrier of T.

Let us consider T. The functor $\mathrm{ClosedPartitions}\, T$ yields a non empty partition family of the carrier of T and is defined by:

(Def. 4) $\mathrm{ClosedPartitions}\, T = \{A; A$ ranges over partitions of the carrier of T: A is closed$\}$.

Let T be a non empty topological space. The functor $\mathrm{T}_1\text{-reflex}\, T$ yields a topological space and is defined as follows:

(Def. 5) $\mathrm{T}_1\text{-reflex}\, T =$ the decomposition space of $\mathrm{Intersection}\, \mathrm{ClosedPartitions}\, T$.

Let us consider T. Note that $\mathrm{T}_1\text{-reflex}\, T$ is strict and non empty.

Next we state the proposition

(12) For every non empty topological space T holds $\mathrm{T}_1\text{-reflex}\, T$ is a T_1 space.

Let T be a non empty topological space. The functor $\mathrm{T}_1\text{-reflect}\, T$ yielding a continuous map from T into $\mathrm{T}_1\text{-reflex}\, T$ is defined as follows:

(Def. 6) T_1-reflect T = the projection onto Intersection ClosedPartitions T.

The following four propositions are true:

(13) Let T, T_1 be non empty topological spaces and f be a continuous map from T into T_1. Suppose T_1 is a T_1 space. Then

(i) $\{f^{-1}(\{z\}); z$ ranges over elements of T_1: $z \in \mathrm{rng}\, f\}$ is a partition of the carrier of T, and

(ii) for every subset A of T such that $A \in \{f^{-1}(\{z\}); z$ ranges over elements of T_1: $z \in \mathrm{rng}\, f\}$ holds A is closed.

(14) Let T, T_1 be non empty topological spaces and f be a continuous map from T into T_1. Suppose T_1 is a T_1 space. Let given w and x be an element of T. If $w = \mathrm{EqClass}(x, \mathrm{Intersection\ ClosedPartitions}\, T)$, then $w \subseteq f^{-1}(\{f(x)\})$.

(15) Let T, T_1 be non empty topological spaces and f be a continuous map from T into T_1. Suppose T_1 is a T_1 space. Let given w. Suppose $w \in$ the carrier of T_1-reflex T. Then there exists an element z of T_1 such that $z \in \mathrm{rng}\, f$ and $w \subseteq f^{-1}(\{z\})$.

(16) Let T, T_1 be non empty topological spaces and f be a continuous map from T into T_1. Suppose T_1 is a T_1 space. Then there exists a continuous map h from T_1-reflex T into T_1 such that $f = h \cdot \mathrm{T}_1\text{-reflect}\, T$.

Let T, S be non empty topological spaces and let f be a continuous map from T into S. The functor T_1-reflex f yields a continuous map from T_1-reflex T into T_1-reflex S and is defined as follows:

(Def. 7) $\mathrm{T}_1\text{-reflect}\, S \cdot f = \mathrm{T}_1\text{-reflex}\, f \cdot \mathrm{T}_1\text{-reflect}\, T$.

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