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Calculation of Matrices of Field Elements. Part I

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Summary. This article gives property of calculation of matrices.

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The articles [8], [3], [10], [11], [4], [1], [5], [2], [13], [6], [7], [12], and [9] provide the notation and terminology for this paper.

In this paper i denotes a natural number.

Let K be a field and let M_1, M_2 be matrices over K. The functor $M_1 - M_2$ yielding a matrix over K is defined by:

(Def. 1) $M_1 - M_2 = M_1 + -M_2$.

One can prove the following propositions:

(1) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $--M = M$.

(2) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $M + -M = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M)\times(\operatorname{width} M)}$.

(3) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $M - M = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M)\times(\operatorname{width} M)}$.

(4) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_3 = M_2 + M_3$. Then $M_1 = M_2$.

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(5) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_2 > 0$ holds $M_1 - -M_2 = M_1 + M_2$.

(6) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 = M_1 + M_2$ holds $M_2 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}$.

(7) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_2 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}$ holds $M_1 = M_2$.

(8) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}$ holds $M_2 = -M_1$.

(9) For all natural numbers n, m and for every field K such that $n > 0$ holds $-\begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{n \times m} = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{n \times m}$.

(10) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_2 - M_1 = M_2$ holds $M_1 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_K^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}$.

(11) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 - (M_2 - M_2)$.

(12) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $-(M_1 + M_2) = -M_1 + -M_2$.

(13) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 - (M_1 - M_2) = M_2$.

(14) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_2 - M_3$. Then $M_1 = M_2$.

(15) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_3 - M_1 = M_3 - M_2$. Then $M_1 = M_2$.

(16) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_2 - M_3 = M_1 - M_3 - M_2$.

(17) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = M_1 - M_2 - (M_3 - M_2)$.

(18) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_3 - M_1 - (M_3 - M_2) = M_2 - M_1$.

(19) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_2 = M_3 - M_4$. Then $M_1 - M_3 = M_2 - M_4$.

(20) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 + (M_2 - M_2)$.

(21) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = (M_1 + M_2) - M_2$.

(22) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = (M_1 - M_2) + M_2$.

(23) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 + M_3 = M_1 + M_2 + (M_3 - M_2)$.

(24) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $(M_1 + M_2) - M_3 = (M_1 - M_3) + M_2$.

(25) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $(M_1 - M_2) + M_3 = (M_3 - M_2) + M_1$.

(26) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 + M_3 = (M_1 + M_2) - (M_2 - M_3)$.

(27) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$

and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = (M_1 + M_2) - (M_3 + M_2)$.

(28) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = M_3 + M_4$. Then $M_1 - M_3 = M_4 - M_2$.

(29) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_4 - M_2$. Then $M_1 + M_2 = M_3 + M_4$.

(30) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = M_3 - M_4$. Then $M_1 + M_4 = M_3 - M_2$.

(31) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 + M_3) = M_1 - M_2 - M_3$.

(32) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 - M_3) = (M_1 - M_2) + M_3$.

(33) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 - M_3) = M_1 + (M_3 - M_2)$.

(34) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = (M_1 - M_2) + (M_2 - M_3)$.

(35) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $-M_1 = -M_2$, then $M_1 = M_2$.

(36) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ and $-M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\operatorname{len} M)\times(\operatorname{width} M)}$ holds $M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\operatorname{len} M)\times(\operatorname{width} M)}$.

(37) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 =$

$\text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ and $M_1 + -M_2 = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\text{len}\, M_1)\times(\text{width}\, M_1)}$ holds $M_1 = M_2$.

(38) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $M_1 = M_1 + M_2 + -M_2$.

(39) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $M_1 = M_1 + (M_2 + -M_2)$.

(40) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $M_1 = -M_2 + M_1 + M_2$.

(41) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $-(-M_1 + M_2) = M_1 + -M_2$.

(42) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $M_1 + M_2 = -(-M_1 + -M_2)$.

(43) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $-(M_1 - M_2) = M_2 - M_1$.

(44) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $-M_1 - M_2 = -M_2 - M_1$.

(45) For every field K and for all matrices M_1, M_2 over K such that $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{len}\, M_1 > 0$ holds $M_1 = -M_2 - (-M_1 - M_2)$.

(46) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{len}\, M_2 = \text{len}\, M_3$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{width}\, M_2 = \text{width}\, M_3$ and $\text{len}\, M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_1 - M_3 - M_2$.

(47) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{len}\, M_2 = \text{len}\, M_3$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{width}\, M_2 = \text{width}\, M_3$ and $\text{len}\, M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_2 - M_3 - M_1$.

(48) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{len}\, M_2 = \text{len}\, M_3$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{width}\, M_2 = \text{width}\, M_3$ and $\text{len}\, M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_3 - M_2 - M_1$.

(49) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\text{len}\, M_1 = \text{len}\, M_2$ and $\text{len}\, M_2 = \text{len}\, M_3$ and $\text{width}\, M_1 = \text{width}\, M_2$ and $\text{width}\, M_2 = \text{width}\, M_3$ and $\text{len}\, M_1 > 0$, then $M_3 - M_1 - (M_3 - M_2) = -(M_1 - M_2)$.

(50) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $\begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\operatorname{len} M)\times(\operatorname{width} M)} - M = -M$.

(51) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 + M_2 = M_1 - -M_2$.

(52) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 - (M_2 + -M_2)$.

(53) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_2 + -M_3$. Then $M_1 = M_2$.

(54) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_3 - M_1 = M_3 + -M_2$. Then $M_1 = M_2$.

(55) Let K be a field and A, B be matrices over K. If $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$, then the indices of A = the indices of B.

(56) Let K be a field and x, y, z be finite sequences of elements of the carrier of K. If $\operatorname{len} x = \operatorname{len} y$ and $\operatorname{len} y = \operatorname{len} z$, then $(x + y) \bullet z = x \bullet z + y \bullet z$.

(57) Let K be a field and x, y, z be finite sequences of elements of the carrier of K. If $\operatorname{len} x = \operatorname{len} y$ and $\operatorname{len} y = \operatorname{len} z$, then $z \bullet (x + y) = z \bullet x + z \bullet y$.

(58) Let D be a non empty set and M be a matrix over D. Suppose $\operatorname{len} M > 0$. Let n be a natural number. Then M is a matrix over D of dimension $n \times \operatorname{width} M$ if and only if $n = \operatorname{len} M$.

(59) Let K be a field, j be a natural number, and A, B be matrices over K. Suppose $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$ and there exists a natural number j such that $\langle i, j\rangle \in$ the indices of A. Then $\operatorname{Line}(A + B, i) = \operatorname{Line}(A, i) + \operatorname{Line}(B, i)$.

(60) Let K be a field, j be a natural number, and A, B be matrices over K. Suppose $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$ and there exists a natural number i such that $\langle i, j\rangle \in$ the indices of A. Then $(A + B)_{\square,j} = A_{\square,j} + B_{\square,j}$.

(61) Let V_1 be a field and P_1, P_2 be finite sequences of elements of the carrier of V_1. If $\operatorname{len} P_1 = \operatorname{len} P_2$, then $\sum(P_1 + P_2) = \sum P_1 + \sum P_2$.

(62) Let K be a field and A, B, C be matrices over K. If $\operatorname{len} B = \operatorname{len} C$ and $\operatorname{width} B = \operatorname{width} C$ and $\operatorname{width} A = \operatorname{len} B$ and $\operatorname{len} A > 0$ and $\operatorname{len} B > 0$, then $A \cdot (B + C) = A \cdot B + A \cdot C$.

(63) Let K be a field and A, B, C be matrices over K. If $\operatorname{len} B = \operatorname{len} C$ and

width B = width C and len A = width B and len $B > 0$ and len $A > 0$, then $(B + C) \cdot A = B \cdot A + C \cdot A$.

(64) Let K be a field, n, m, k be natural numbers, M_1 be a matrix over K of dimension $n \times m$, and M_2 be a matrix over K of dimension $m \times k$. Suppose width M_1 = len M_2 and $0 <$ len M_1 and $0 <$ len M_2. Then $M_1 \cdot M_2$ is a matrix over K of dimension $n \times k$.

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