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Partial Sum of Some Series

Ming Liang
QingDao QiuShi College
of Vocation and Technology

Yuzhong Ding
QingDao University
of Science and Technology

Summary. Solving the partial sum of some often used series.

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The articles [2], [1], [4], [3], [5], [7], and [6] provide the notation and terminology for this paper.

In this paper n is a natural number and s is a sequence of real numbers.

Next we state a number of propositions:

(1) $|(-1)^n| = 1$.

(2) $(n+1)^3 = n^3+3\cdot n^2+3\cdot n+1$ and $(n+1)^4 = n^4+4\cdot n^3+6\cdot n^2+4\cdot n+1$ and $(n+1)^5 = n^5+5\cdot n^4+10\cdot n^3+10\cdot n^2+5\cdot n+1$.

(3) If for every n holds $s(n) = n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)}{2}$.

(4) If for every n holds $s(n) = 2\cdot n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = n\cdot(n+1)$.

(5) If for every n holds $s(n) = 2\cdot n+1$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (n+1)^2$.

(6) If for every n holds $s(n) = n\cdot(n+1)$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(n+2)}{3}$.

(7) If for every n holds $s(n) = n\cdot(n+1)\cdot(n+2)$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(n+2)\cdot(n+3)}{4}$.

(8) If for every n holds $s(n) = n\cdot(n+1)\cdot(n+2)\cdot(n+3)$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(n+2)\cdot(n+3)\cdot(n+4)}{5}$.

(9) If for every n holds $s(n) = \frac{1}{n\cdot(n+1)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = 1-\frac{1}{n+1}$.

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(10) If for every n holds $s(n) = \frac{1}{n\cdot(n+1)\cdot(n+2)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{1}{4} - \frac{1}{2\cdot(n+1)\cdot(n+2)}$.

(11) If for every n holds $s(n) = \frac{1}{n\cdot(n+1)\cdot(n+2)\cdot(n+3)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{1}{18} - \frac{1}{3\cdot(n+1)\cdot(n+2)\cdot(n+3)}$.

(12) If for every n holds $s(n) = n^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(2\cdot n+1)}{6}$.

(13) If for every n holds $s(n) = (-1)^{n+1} \cdot n^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{(-1)^{n+1}\cdot n\cdot(n+1)}{2}$.

(14) If for every n such that $n \geq 1$ holds $s(n) = (2 \cdot n - 1)^2$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(4\cdot n^2-1)}{3}$.

(15) If for every n holds $s(n) = n^3$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n^2\cdot(n+1)^2}{4}$.

(16) If for every n such that $n \geq 1$ holds $s(n) = (2\cdot n-1)^3$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = n^2 \cdot (2 \cdot n^2 - 1)$.

(17) If for every n holds $s(n) = n^4$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(2\cdot n+1)\cdot((3\cdot n^2+3\cdot n)-1)}{30}$.

(18) If for every n holds $s(n) = (-1)^{n+1} \cdot n^4$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{(-1)^{n+1}\cdot n\cdot(n+1)\cdot((n^2+n)-1)}{2}$.

(19) If for every n holds $s(n) = n^5$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n^2\cdot(n+1)^2\cdot((2\cdot n^2+2\cdot n)-1)}{12}$.

(20) If for every n holds $s(n) = n^6$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(2\cdot n+1)\cdot(((3\cdot n^4+6\cdot n^3)-3\cdot n)+1)}{42}$.

(21) If for every n holds $s(n) = n^7$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n^2\cdot(n+1)^2\cdot(((3\cdot n^4+6\cdot n^3)-n^2-4\cdot n)+2)}{24}$.

(22) If for every n holds $s(n) = n \cdot (n + 1)^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(n+2)\cdot(3\cdot n+5)}{12}$.

(23) If for every n holds $s(n) = n \cdot (n + 1)^2 \cdot (n + 2)$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n\cdot(n+1)\cdot(n+2)\cdot(n+3)\cdot(2\cdot n+3)}{10}$.

(24) If for every n holds $s(n) = n \cdot (n + 1) \cdot 2^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = 2^{n+1} \cdot ((n^2 - n) + 2) - 4$.

(25) Suppose that for every n such that $n \geq 2$ holds $s(n) = \frac{1}{(n-1)\cdot(n+1)}$ and $s(0) = 0$ and $s(1) = 0$. Let given n. If $n \geq 2$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{3}{4} - \frac{1}{2\cdot n} - \frac{1}{2\cdot(n+1)}$.

(26) If for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(2\cdot n-1)\cdot(2\cdot n+1)}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n}{2\cdot n+1}$.

(27) If for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(3\cdot n-2)\cdot(3\cdot n+1)}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{n}{3\cdot n+1}$.

(28) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(2 \cdot n - 1) \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 3)}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{12} - \frac{1}{4 \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 3)}$.

(29) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(3 \cdot n - 2) \cdot (3 \cdot n + 1) \cdot (3 \cdot n + 4)}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{24} - \frac{1}{6 \cdot (3 \cdot n + 1) \cdot (3 \cdot n + 4)}$.

(30) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{2 \cdot n - 1}{n \cdot (n+1) \cdot (n+2)}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = (\frac{3}{4} - \frac{2}{n+2}) + \frac{1}{2 \cdot (n+1) \cdot (n+2)}$.

(31) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{n+2}{n \cdot (n+1) \cdot (n+3)}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{29}{36} - \frac{1}{n+3} - \frac{3}{2 \cdot (n+2) \cdot (n+3)} - \frac{4}{3 \cdot (n+1) \cdot (n+2) \cdot (n+3)}$.

(32) If for every n holds $s(n) = \frac{(n+1) \cdot 2^n}{(n+2) \cdot (n+3)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{2^{n+1}}{n+3} - \frac{1}{2}$.

(33) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{n^2 \cdot 4^n}{(n+1) \cdot (n+2)}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{2}{3} + \frac{(n-1) \cdot 4^{n+1}}{3 \cdot (n+2)}$.

(34) If for every n such that $n \geq 1$ holds $s(n) = \frac{n+2}{n \cdot (n+1) \cdot 2^n}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n+1) \cdot 2^n}$.

(35) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{2 \cdot n + 3}{n \cdot (n+1) \cdot 3^n}$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n+1) \cdot 3^n}$.

(36) If for every n holds $s(n) = \frac{(-1)^n \cdot 2^{n+1}}{(2^{n+1} + (-1)^{n+1}) \cdot (2^{n+2} + (-1)^{n+2})}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{3} + \frac{(-1)^{n+2}}{3 \cdot (2^{n+2} + (-1)^{n+2})}$.

(37) If for every n holds $s(n) = n! \cdot n$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = (n+1)! - 1$.

(38) If for every n holds $s(n) = \frac{n}{(n+1)!}$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n+1)!}$.

(39) If for every n such that $n \geq 1$ holds $s(n) = \frac{(n^2+n)-1}{(n+2)!}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{2} - \frac{n+1}{(n+2)!}$.

(40) If for every n such that $n \geq 1$ holds $s(n) = \frac{n \cdot 2^n}{(n+2)!}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{2^{n+1}}{(n+2)!}$.

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