FORMALIZED MATHEMATICS
Volume 13, Number 4, Pages 537–540
University of Białystok, 2005

# On the Calculus of Binary Arithmetics. Part II

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**Summary.** In this paper, we introduce binary arithmetic and its related operations. We include some theorems concerning logical operators.

MML identifier: BINARI_6, version: 7.6.01 4.50.934

The terminology and notation used in this paper are introduced in the following articles: [4], [3], [2], and [1].

In this paper $x$, $y$, $z$ denote boolean sets.

Next we state a number of propositions:

(1) $\mathit{true} \Rightarrow x = x$.

(2) $\mathit{false} \Rightarrow x = \mathit{true}$.

(3) $x \Rightarrow x = \mathit{true}$ and $\neg(x \Rightarrow x) = \mathit{false}$.

(4) $\neg(x \Rightarrow y) = x \wedge \neg y$.

(5) $x \Rightarrow \neg x = \neg x$ and $\neg(x \Rightarrow \neg x) = x$.

(6) $\neg x \Rightarrow x = x$.

(7) $\mathit{true} \Leftrightarrow x = x$.

(8) $\mathit{false} \Leftrightarrow x = \neg x$.

(9) $x \Leftrightarrow x = \mathit{true}$ and $\neg(x \Leftrightarrow x) = \mathit{false}$.

(10) $\neg x \Leftrightarrow x = \mathit{false}$.

(11) $x \wedge (y \Leftrightarrow z) = x \wedge (\neg y \vee z) \wedge (\neg z \vee y)$.

(12) $x \wedge (y \ \mathrm{'nand'}\ z) = x \wedge \neg y \vee x \wedge \neg z$.

(13) $x \wedge (y \ \mathrm{'nor'}\ z) = x \wedge \neg y \wedge \neg z$.

(14) $x \wedge (x \wedge y) = x \wedge y$.




ISSN 1426–2630

(15) $x \wedge (x \vee y) = x \vee x \wedge y.$

(16) $x \wedge (x \oplus y) = x \wedge \neg y.$

(17) $x \wedge (x \Rightarrow y) = x \wedge y.$

(18) $x \wedge (x \Leftrightarrow y) = x \wedge y.$

(19) $x \wedge (x\ '\mathrm{nand}'\ y) = x \wedge \neg y.$

(20) $x \wedge (x\ '\mathrm{nor}'\ y) = \mathit{false}.$

(21) $x \vee (y \oplus z) = x \vee \neg y \wedge z \vee y \wedge \neg z.$

(22) $x \vee (y \Leftrightarrow z) = (x \vee \neg y \vee z) \wedge (x \vee \neg z \vee y).$

(23) $x \vee (y\ '\mathrm{nand}'\ z) = x \vee \neg y \vee \neg z.$

(24) $x \vee (y\ '\mathrm{nor}'\ z) = (x \vee \neg y) \wedge (x \vee \neg z)$ and $x \vee (y\ '\mathrm{nor}'\ z) = (y \Rightarrow x) \wedge (z \Rightarrow x).$

(25) $x \vee (x \vee y) = x \vee y.$

(26) $x \vee (x \Rightarrow y) = \mathit{true}.$

(27) $x \vee (x \Leftrightarrow y) = y \Rightarrow x.$

(28) $x \vee (x\ '\mathrm{nand}'\ y) = \mathit{true}.$

(29) $x \vee (x\ '\mathrm{nor}'\ y) = y \Rightarrow x.$

(30) $x \Rightarrow y \oplus z = \neg x \vee \neg y \wedge z \vee y \wedge \neg z.$

(31) $x \Rightarrow y \Leftrightarrow z = (\neg x \vee \neg y \vee z) \wedge (\neg x \vee y \vee \neg z).$

(32) $x \Rightarrow y\ '\mathrm{nand}'\ z = \neg x \vee \neg y \vee \neg z.$

(33) $x \Rightarrow y\ '\mathrm{nor}'\ z = (\neg x \vee \neg y) \wedge (\neg x \vee \neg z)$ and $x \Rightarrow y\ '\mathrm{nor}'\ z = (x \Rightarrow \neg y) \wedge (x \Rightarrow \neg z).$

(34) $x \Rightarrow x \wedge y = x \Rightarrow y.$

(35) $x \Rightarrow x \vee y = \mathit{true}.$

(36) $x \Rightarrow x \oplus y = \neg x \vee \neg y.$

(37) $x \Rightarrow x \Rightarrow y = x \Rightarrow y.$

(38) $x \Rightarrow x \Leftrightarrow y = x \Rightarrow y$ and $x \Rightarrow x \Leftrightarrow y = x \Rightarrow x \Rightarrow y.$

(39) $x \Rightarrow x\ '\mathrm{nand}'\ y = \neg(x \wedge y).$

(40) $x \Rightarrow x\ '\mathrm{nor}'\ y = \neg x.$

(41) $x\ '\mathrm{nand}'\ (y \Rightarrow z) = (\neg x \vee y) \wedge (\neg x \vee \neg z)$ and $x\ '\mathrm{nand}'\ (y \Rightarrow z) = (x \Rightarrow y) \wedge (x \Rightarrow \neg z).$

(42) $x\ '\mathrm{nand}'\ (y \Leftrightarrow z) = \neg(x \wedge (\neg y \vee z) \wedge (\neg z \vee y)).$

(43) $x\ '\mathrm{nand}'\ (y\ '\mathrm{nand}'\ z) = (\neg x \vee y) \wedge (\neg x \vee z)$ and $x\ '\mathrm{nand}'\ (y\ '\mathrm{nand}'\ z) = (x \Rightarrow y) \wedge (x \Rightarrow z).$

(44) $x\ '\mathrm{nand}'\ (y\ '\mathrm{nor}'\ z) = \neg x \vee y \vee z.$

(45) $x\ '\mathrm{nand}'\ x \wedge y = \neg(x \wedge y).$

(46) $x\ '\mathrm{nand}'\ (x \oplus y) = x \Rightarrow y.$

(47) $x\ '\mathrm{nand}'\ (x \Rightarrow y) = \neg(x \wedge y).$

(48) $x\ '\mathrm{nand}'\ (x \Leftrightarrow y) = \neg(x \wedge y).$

(49) $x\ \text{'nand'}\ (x\ \text{'nand'}\ y) = x \Rightarrow y.$

(50) $x\ \text{'nand'}\ (x\ \text{'nor'}\ y) = \mathit{true}.$

(51) $x\ \text{'nor'}\ (y \oplus z) = \neg(x \vee \neg y \wedge z \vee y \wedge \neg z).$

(52) $x\ \text{'nor'}\ (y \Leftrightarrow z) = \neg((x \vee \neg y \vee z) \wedge (x \vee \neg z \vee y)).$

(53) $x\ \text{'nor'}\ (y\ \text{'nand'}\ z) = \neg x \wedge y \wedge z.$

(54) $x\ \text{'nor'}\ (y\ \text{'nor'}\ z) = \neg x \wedge y \vee \neg x \wedge z.$

(55) $x\ \text{'nor'}\ x \wedge y = \neg x.$

(56) $x\ \text{'nor'}\ (x \vee y) = \neg x \wedge \neg y.$

(57) $x\ \text{'nor'}\ (x \oplus y) = \neg x \wedge \neg y.$

(58) $x\ \text{'nor'}\ (x \Rightarrow y) = \mathit{false}.$

(59) $x\ \text{'nor'}\ (x \Leftrightarrow y) = \neg x \wedge y.$

(60) $x\ \text{'nor'}\ (x\ \text{'nand'}\ y) = \mathit{false}.$

(61) $x\ \text{'nor'}\ (x\ \text{'nor'}\ y) = \neg x \wedge y.$

(62) $x \oplus y \wedge z = (x \vee y \wedge z) \wedge (\neg x \vee \neg(y \wedge z)).$

(63) $x \oplus x \wedge y = x \wedge \neg y.$

(64) $x \oplus (x \vee y) = \neg x \wedge y.$

(65) $\neg x \wedge (x \oplus y) = \neg x \wedge y.$

(66) $x \wedge \neg(x \oplus y) = x \wedge y.$

(67) $x \oplus (x \oplus y) = y.$

(68) $x \wedge \neg(x \Rightarrow y) = x \wedge \neg y.$

(69) $x \oplus (x \Rightarrow y) = \neg x \vee \neg y.$

(70) $\neg x \wedge (x \Leftrightarrow y) = \neg x \wedge \neg y.$

(71) $x \wedge \neg(x \Leftrightarrow y) = x \wedge \neg y.$

(72) $x \oplus (x \Leftrightarrow y) = \neg y.$

(73) $x \oplus (x\ \text{'nand'}\ y) = x \Rightarrow y.$

(74) $x \oplus (x\ \text{'nor'}\ y) = y \Rightarrow x.$

(75) $\neg x \wedge (x \Rightarrow y) = \neg x \vee \neg x \wedge y.$

(76) $\neg x \wedge (y \Leftrightarrow z) = \neg x \wedge (\neg y \vee z) \wedge (\neg z \vee y).$

(77) $\neg x \wedge (x \Leftrightarrow y) = \neg x \wedge \neg y \wedge (\neg x \vee y).$

(78) $\neg x \wedge (x\ \text{'nand'}\ y) = \neg x \vee \neg x \wedge \neg y.$

(79) $\neg x \wedge (x\ \text{'nor'}\ y) = \neg x \wedge \neg y.$

(80) $\neg x \vee (x \Rightarrow y) = \neg x \vee y.$

(81) $\neg x \vee (x \Leftrightarrow y) = \neg x \vee y.$

(82) $\neg x \vee (x\ \text{'nand'}\ y) = \neg x \vee \neg y.$

(83) $\neg x \oplus (x \Rightarrow y) = x \wedge y.$

(84) $\neg x \oplus (y \Rightarrow x) = x \wedge (x \vee \neg y) \vee \neg x \wedge y.$

(85) $\neg(x \Rightarrow y) = x \wedge \neg y.$

(86) $\neg(x \Leftrightarrow y) = x \wedge \neg y \vee y \wedge \neg x$.

(87) $\neg x \oplus (x \Leftrightarrow y) = y$.

## References

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*Received November 23, 2005*